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COMPLETE PERSPECTIVE COURSE

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PERSPECTIVE COURSE

COMPRISING

THE ELEMENTARY AND ADVANCED STAGES OF PERSPECTIVE
THE PROJECTION OF SHADOWS AND REFLECTIONS
WITH EXERCISES IN THEORY AND PRACTICE

ALSO THE

PRACTICAL APPLICATION OF PERSPECTIVE

BY

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IN H.M.S. *BRITANNIA*

*DESIGNED TO MEET THE FOLLOWING REQUIREMENTS:
THE EXAMINATIONS OF THE SCIENCE AND ART DEPARTMENT, SOUTH
KENSINGTON, AND OF THE COLLEGE OF PRECEPTORS; THE
OXFORD AND CAMBRIDGE LOCAL EXAMINATIONS*

ALSO FOR

THE USE OF ARCHITECTS, ENGINEERS, ARTISTS, AND ART STUDENTS

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PREFACE

THE Author, in introducing this work to the public, feels that, notwithstanding the many books on the subject already existing, there is still room for one that comprises the whole course of Perspective treated in a comprehensive manner, with the allied subjects of Projection of Shadows and Reflections.

The Art of Perspective is one in which the Author has had many years' experience, both as a teacher as well as in its practical application; he consequently feels that he is not unqualified to deal with this subject in all its branches.

The design of the work is to provide information on Perspective to every class; to those who require the rudiments of the subject, as well as to those who desire a complete course of instruction. There are many students desirous of obtaining sufficient knowledge of Perspective to enable them to sketch correctly, and to test the accuracy of their drawings, but at the same time with a strong aversion to plodding through a series of dry problems to obtain this information. To meet the requirements of this class of students, the whole subject has been explained in the form of an introduction in simple language, with illustrations from familiar objects—the elementary portion in Chapter II.; and the more advanced part of

the subject in Chapter IX. If these chapters are carefully studied, they will impart sufficient knowledge of the principles of the subject to enable them to utilise the laws of perspective.

Those, on the other hand, who require a thorough course will find it in Chapters II.-XI.

Another portion of the work is devoted to the practical application of perspective, and this the Author hopes will be found useful to Artists as well as to Students. The greater part of this information is the outcome of experience gained during many years of professional work; and it includes Aërial Perspective, Chapter XIII.; Perspective Hints for Artists, Chapter XIV.; Perspective for Architects and Engineers, with the use of the Centrolinead, Chapter XV.; and Photographic Perspective, introducing a method for correcting the exaggerated perspective of photographs, Chapter XVI.

The book contains all the necessary information for the Science and Art Examinations—the Elementary Stage in Chapters II.-VII.; and the Advanced Stage in Chapters IX.-XI. It also covers the ground of the Oxford and Cambridge Local Examinations, and of those of the College of Preceptors.

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COMPLETE PERSPECTIVE COURSE

CHAPTER I

GEOMETRICAL DEFINITIONS

A *point* simply marks a position; it is supposed to have no magnitude.

A *line* has length without breadth or thickness. The extremities and intersections of lines are points. A *straight* line or *right* line is one that is in the same direction throughout its length, and is the shortest that can be drawn between two points. To *produce* a line is to lengthen it.

A *plane* is a flat even surface; it has length and breadth only. The intersections of planes are straight lines.

Parallel lines are such as are in the same plane, and at equal distances apart throughout their entire length; if produced they would never meet (Fig. 1).

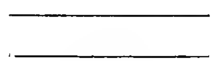


Fig. 1

A *circle* is a plane figure bounded by a curved line, such that all straight lines drawn to it from a certain point are equal. This point is called the *centre*, and the curved line is called the *circumference* of the circle.

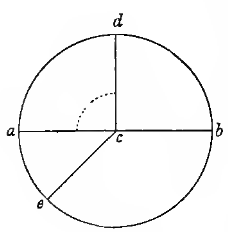


Fig. 2

A straight line drawn from the centre to the circumference, as *ce* or *cd* (Fig. 2) is called a *radius*. A straight line drawn

through the centre, and terminated at both ends by the circumference, as *ab*, is called a *diameter*. A *semicircle* is half a circle, as *adbc*. A *quadrant* is a quarter of a circle, as *adc*.

An *arc* is any portion of the circumference of a circle, as *abc* (Fig. 3). A *chord* is a straight line joining the extremities of an arc, as *ac*. A *segment* is the space enclosed by the arc and its chord, as *f*. A *sector* is the space enclosed by two radii and the arc between them, as *g*. A *tangent* is a line touching the circumference, as *de*; it is always at right angles to the radius of the circle at the point of contact.

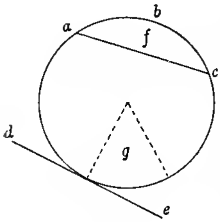


Fig. 3

An *angle* is the inclination of two straight lines meeting in a point. This point is called the *vertex* of the angle, as *a* (Fig. 4). The angle here shown would be called either *bac* or *cab*.

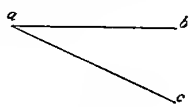


Fig. 4

If two adjacent angles made by two straight lines at the point where they meet be equal, as *dca* and *dcb* (Fig. 2), each of these angles is called a right angle, and either of the straight lines may be said to be perpendicular to the other.

A *right angle* is supposed to be divided into 90 equal parts, each of which is called a *degree*. A degree is expressed in writing by a small circle placed over the last figure of the numerals denoting the number of degrees—thus 36° means thirty-six degrees.

The circumference of a circle is supposed to be divided into 360 equal arcs, each of which subtends an angle of 1°. Sometimes this *arc* is itself loosely termed a *degree*.

An angle containing more than 90° is called an *obtuse angle*, as *ccb* (Fig. 2); while an angle containing less than 90° is called an *acute angle*, as *ace*.

A line is said to be perpendicular to a plane when it is at right angles to any straight line in that plane.

Concentric circles have the same centre.

Triangles

Triangles are closed figures contained by three straight lines.

A triangle which has all its sides equal is called *equilateral* (Fig. 5).

N.B.—Such a triangle will always have its three angles equal, and therefore will also be *equiangular.*

A triangle which has two sides (and therefore two angles) equal is called *isosceles* (Fig. 6) (*isos,* equal; *skelos,* a leg).

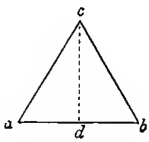


Fig. 5

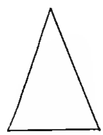


Fig. 6

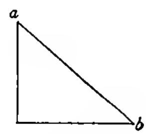


Fig. 7

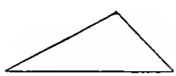


Fig. 8

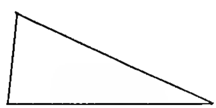


Fig. 9

A triangle which has a right angle is called *right-angled* (Fig. 7). The side opposite to the right angle, as *ab,* is called the *hypotenuse* (*hypo,* under; *tenein,* to stretch).

A triangle which has an obtuse angle is called *obtuse-angled* (Fig. 8).

A triangle which has three acute angles is called *acute-angled* (Fig. 9).

The *base* of a triangle is its lowest side, as *ab* (Fig. 5).

The *vertex* is the point opposite the base, as *c* (Fig. 5).

The *altitude* or perpendicular height is a line drawn from the vertex at right angles to its base, as *cd* (Fig. 5).

Quadrilateral Figures

Quadrilateral figures are such as are bounded by four straight lines.

A quadrilateral figure which has four equal sides is called a *rhombus* (Fig. 10)—Gk. *rhombos,* from *rhembein,* to twirl, from some likeness to a spindle.

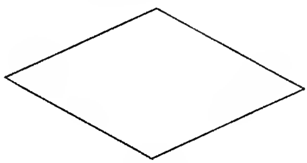


Fig. 10

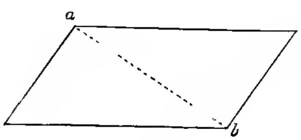


Fig. 11

A quadrilateral figure whose opposite sides are parallel is called a *parallelogram* (Fig. 11).

A parallelogram whose angles are right angles is called a *rectangle* (Fig. 12).

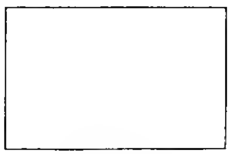


Fig. 12

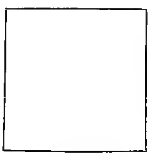


Fig. 13

A rectangle which has its sides equal is called a *square* (Fig. 13).

A line joining two opposite angles of a quadrilateral figure is called a *diagonal,* as the dotted line *ab* (Fig. 11).

Polygons

A *polygon* is a plane figure which has more than four angles.

A polygon which is both equilateral and equiangular is called *regular.*

A polygon	of five	sides	is called	a	*pentagon.*
,,	,,	six	,,	,,	*hexagon.*
,,	,,	seven	,,	,,	*heptagon.*
,,	,,	eight	,,	,,	an *octagon,* etc.

Solids

A *solid* has length, breadth, and thickness. A solid bounded wholly by planes is called a *polyhedron* (*poly,* many; *hedra,* a side).

A solid bounded by six planes or faces, whereof the opposite ones are parallel, is called a *parallelepiped* (*parallelos,* parallel; and *epipedon,* a plane).

A parallelepiped whose angles are all right angles is called a *rectangular parallelepiped* or *orthohedron* (*orthos,* right; and *hedra,* a side).

An orthohedron with six equal faces is called a *cube* (*kubos,* a die).

A polyhedron, all but one of whose faces meet in a point, is called a *pyramid* (Gk. *pyramis,* a pyramid).

Pyramids are often named, after the shape of their bases, *triangular, square,* etc.

A polyhedron, all but two of whose faces are parallel to one straight line, is called a *prism* (Gk. *prisma,* from *prizein,* to saw, a portion sawn off).

If the ends of a prism are at right angles to the straight line to which the other faces are parallel it is called a *right* prism.

Prisms are often named, after the shape of their ends, *triangular, hexagonal,* etc.

A *cylinder* is a solid described by the revolution of a rectangle about one of its sides which remains fixed. This fixed line is called the *axis* of the cylinder.

A right circular cone—generally spoken of simply as a *cone*—is a solid described by the revolution of a right-angled triangle

about one of the sides containing the right angle, which side remains fixed. This fixed line is called the *axis* of the cone; the base is a circle, and the point opposite the base is called the *vertex*.

A solid bounded by a closed surface, such that all straight lines drawn to it from a certain point are equal, is called a *sphere* (Gk. *sphaira*, a ball).

The point referred to is called the *centre* of the sphere.

Technical Definitions

A plane parallel to the ground, or, more strictly speaking, parallel to still water, is called a *horizontal* plane.

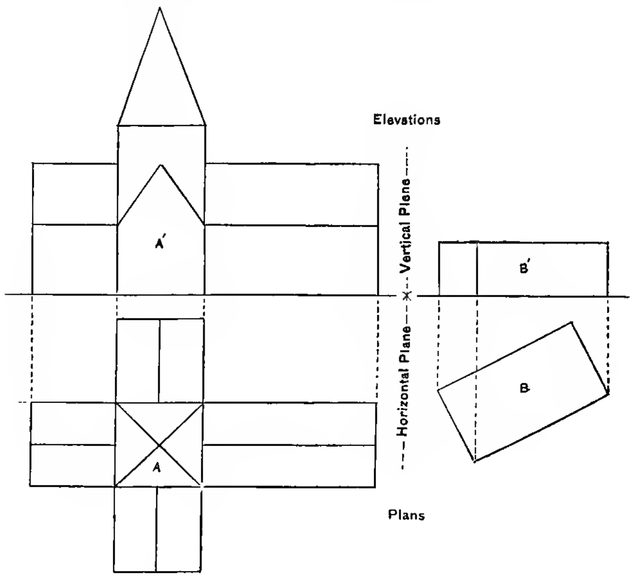


Fig. 14

A *vertical* plane is a plane at right angles to a horizontal plane.

A *horizontal* line is a line parallel to a horizontal plane.

A *vertical* line is a line at right angles to a horizontal plane.

A *plan* of an object is the tracing made on a horizontal plane by the foot of a vertical line, which moves so as to pass successively through the various points and outlines of the object, as A and B (Fig. 14).

An *elevation* of an object is the tracing made on a vertical plane by the end of a horizontal line, at right angles to the vertical plane, which moves so as to pass successively through the various points and outlines of the object, as A′ and B′ (Fig. 14).

N.B.—A is an object parallel to the vertical plane, and B an object inclined to it.

CHAPTER II

INTRODUCTION

1. Perspective (from *perspicio,* I see through) treats of the appearances of objects, and in this differs from geometry, which treats of their actual shapes.

It is the art of representing on a plane—a sheet of paper for instance—objects as they appear to the eye.

The plane on which the object is drawn may be considered as transparent, and we can imagine that we are looking through it at the object represented.

If on looking at an object through a sheet of glass we were to trace its outline on the glass, the figure drawn would be in true linear perspective.

There are two kinds of perspective, viz. *linear* and *aërial.*

Linear perspective relates to the apparent form or outline of objects.

Aërial perspective relates to the tone, colour, and distinctness. (This branch of the subject will be treated of in Chapter XIII.)

2. The Ground Plane.—The spectator is assumed to be standing on level ground—that is, on a horizontal plane. This plane, which is supposed to extend in all directions, to infinity, is called the *ground plane.*

3. The Picture Plane.—In Fig. 15 a large transparent plane is represented in a vertical position standing on the ground plane; this is called the *picture plane:* a cube A is

placed on one side of it, and the spectator on the other; *a* is the tracing of the cube on the picture plane.

4. THE GROUND LINE.—The line in which the picture plane meets the ground plane is called the *ground line;* it is the boundary, towards the spectator, of the ground plane in the picture; it is also called the *base line* (Fig. 15).

If the object were placed touching the picture plane, and then traced upon it, the perspective representation of lines in contact with it would be equal in length to the lines themselves.

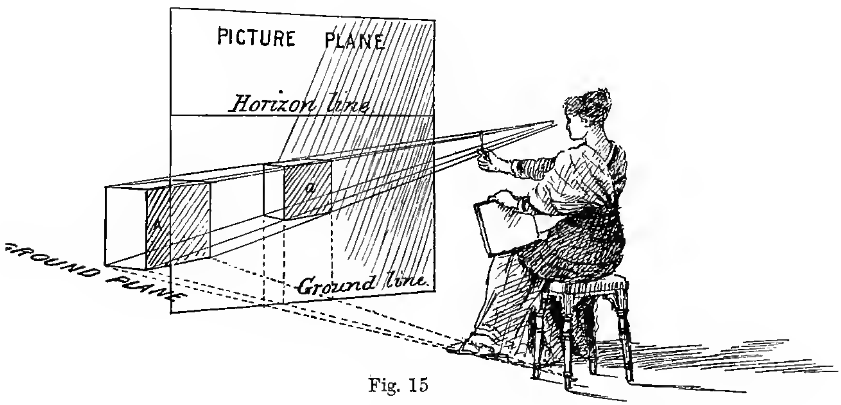


Fig. 15

The farther the object is removed from the picture plane the smaller will be its representation.

When we are drawing an object in perspective, we are supposed to be copying to a given scale the tracing on the picture plane. The picture plane does not really exist; it is assumed to occupy a certain position, which must be determined before we commence our drawing, as the size of the representation of the object depends entirely upon the position in which we place it. The bottom of our drawing represents the ground line.

5. *The picture plane may be defined as an imaginary transparent plane usually placed in a vertical position between the spectator and the object, on which the object is supposed to be traced.*

6. The picture plane is shown in Fig. 15 as a rectangle, but we can imagine it to extend indefinitely in any direction.

7. *Objects of the same magnitude appear to diminish in size as they recede from the eye.*

Let us take two books of the same size, and place them on

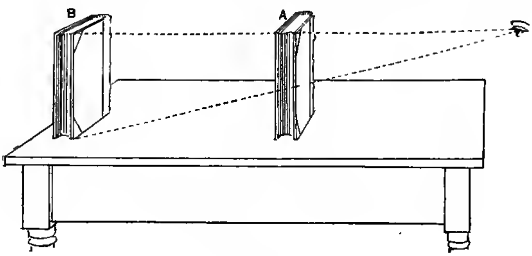


Fig. 16

a table, as shown in Fig. 16. Let the book B be twice as far from the eye as book A. Keep the eye level with their tops, then the lower corner of B will appear to cut halfway up the total height of A.

B being twice as far from the eye as A, appears only half its height; if it were three times as far it would appear one-third of its height, and so on. Consequently, if objects appear to get smaller, parallel lines must appear to get closer together as they recede from the eye, and in the extreme distance would appear to meet.

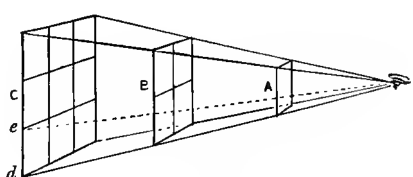


Fig. 17

Instead of taking objects of the same size, let B (Fig. 17) be four times the area of A, and C nine times, and let their distance apart be equal to the distance A is from the eye. If

they were placed in the positions shown, all their corresponding corners would be on the same straight lines; *de* is the same height as A, but its apparent height is only one-third of A. We shall now understand the following :—

8. *The apparent area of an object is inversely as the square of its distance from the eye.*

This is a very important law, for if the size of an object is known, its distance can be determined from its perspective representation. We shall revert to this in Chapter VIII.

9. VISUAL RAYS.—We see an object by means of light dispersed from its surface. Taking a point on the object—a corner, for instance—the light from this point to the eye comes in a straight line, and is called a *visual ray.*

Where the visual ray intersects the picture plane is the point in the picture corresponding to the corner.

By intersecting the visual rays by a plane, we form the picture; when we measure the apparent size of an object with our pencil, we are measuring the distance apart of the visual rays at that distance from the eye (Fig. 15); when we trace an object on glass, we are joining the intersections by the glass of the visual rays as they pass through it.

10. The fundamental problem of perspective is to put a given point into perspective, or, in other words, to find where a given line intersects a given plane. The visual ray from a given point to the eye is the given line, and the picture plane is the given plane. Now any two points in a straight line will give us its direction, and as plane figures may contain or be bounded by straight lines, their position can also be determined by the position of points, and from planes we can determine the place in the picture of the solids that are bounded by those planes.

Before proceeding with problems for the finding of points, lines, etc., we will take a few familiar objects to illustrate the laws of perspective, as by this means the student will become

better acquainted with the terms used; he will also become more readily accustomed to the perspective appearance of lines running in various directions, when they form parts of solids, than when they are taken individually.

The perception of form by the untrained eye is generally inexact, because the actual shape of an object is essentially different from its appearance. We shall understand this better by referring to Fig. 18.

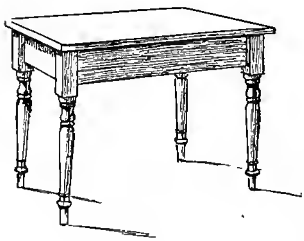


Fig. 18

We know that all four corners of the table top are right angles, yet in the perspective drawing they are all different from each other, and all very unlike right angles. This is why drawing from the model is so difficult for beginners; their knowledge of its true shape interferes with their judgment of its appearance.

One eye only is supposed to be used in perspective; it is clearly impossible without confusion to represent at once, on a plane, an object as it would appear to both eyes. Hold up a pencil at arm's length, in a room, close the left eye, and notice what part of the opposite wall it covers; keep the pencil in the same position, and then look at it with the left eye only; it will now appear to cover an entirely different place. It would be absurd to draw the wall with the pencil in front of it as seen by both eyes at the same time, because the pencil would occupy a different position on the wall for each eye. If we stand before a picture, in the right position, and look at it attentively for some little time, with one eye only, it will appear to be in relief. In a stereoscope we have a picture for each eye, which gives us an impression of solidity.

11. Station Point.—The position of the eye is called the *station point.*

A very useful contrivance for acquiring and proving the laws of perspective is shown in Fig. 19. It consists of a piece

of plate glass about 18 inches × 12 inches, fixed in an upright position on a horizontal board by means of supports, as shown; it has also a piece of stout wire about 6 inches long, bent into a circle at one end, to fix the position of the eye, and a few

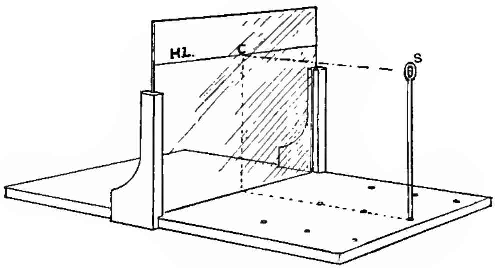


Fig. 19

holes made in the horizontal board, so that the position of the eye can be changed.

It will be advisable to have two or three of these pieces of wire of different lengths, so as to vary the height of the eye; iron meat-skewers will answer the purpose.

If we take a small box and place it with one of its ends

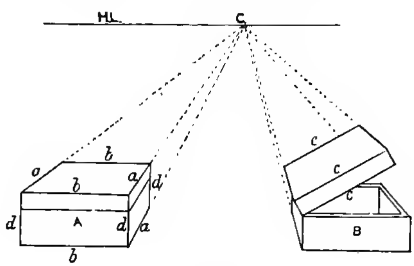


Fig. 20

close to the glass, on the left of the spectator, and trace it as seen from the point S (the station point), its representation on the glass will appear as A (Fig. 20). (A piece of very old soap, cut into strips, is the best material for marking on glass; the tracing can be washed out with a wet sponge.)

Now shift the box to the right of the spectator, and raise the lid a little, still keeping one end close to the glass; when traced in this position, it will appear as shown by B (Fig. 20).

We have now the representation *a*, *a*, *a*, of several lines which are all parallel to one another, and at right angles to the picture plane. If we produce these lines on the glass, we find that they all converge towards one point C; at this point they all meet, and the distances between them vanish. We conclude from this experiment that all the lines which represent those edges of any model, which are at right angles to the glass, will disappear or vanish from sight in the same point C. If they went farther they must diverge, and the distance between them would appear larger, which is absurd. Therefore the lines appear actually to vanish at this point, which is called the *vanishing point* for this series of parallel lines.

12. All lines drawn in perspective may be divided into two kinds, viz. those receding from the picture plane, and those parallel to it.

13. VANISHING POINTS.—We have already proved, by

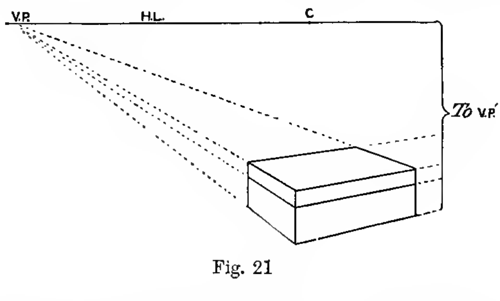


Fig. 21

tracing them on our glass model of the picture plane, that representations of all lines at right angles to that plane meet, if produced, in the same point. Now shift the box, so that the receding lines are no longer at right angles to the glass, and trace them (Fig. 21). Here again it will be seen that the tracings of the receding lines that are parallel to one another

meet in a point; consequently, we infer that the *perspective representations of any set of parallel receding lines will, if produced, meet in a point.* Such a point is called the *vanishing point,* corresponding to that system of parallel lines.

Note.—Numerals in brackets [ ] refer to the numbered articles in this chapter.

A point P is seen in the far distance; a ray of light from this point to the eye intersects the picture plane in a point p [10]. All lines parallel to this ray of light will appear to converge towards the point P, because their distances from the ray, as seen in perspective, gradually diminish until they vanish. As the point p on the picture plane is a tracing of the point P in the distance, all the lines parallel to the ray, drawn on the picture plane, must converge towards p. Then p is called the vanishing point for these lines; and a line from the eye to p, the *vanishing parallel* for all lines parallel to it.

14. POINT OF SIGHT OR CENTRE OF VISION.—The point C (Fig. 20) is called the *point of sight* or *centre of vision,* because it is directly opposite the eye of the spectator, or station point S (Fig. 19); it is the vanishing point for all lines at right angles to the picture plane.

15. DIRECTION OF SIGHT.—The line joining the station point S to the centre of vision C is called the *direction of sight,* or *line of direction;* it may be said to be coincident with the *central* or *principal visual ray.*

Note.—In the examination papers of the Science and Art Department it is called the *line of direction,* but as this term is likely to be confounded by the student with the *directing line,* which will be described later [article 31], we shall call it the *direction of sight.*

16. *In whatever direction the spectator is looking, the picture plane is assumed to be always at right angles to the direction of sight.* The student should bear in mind that the eye is supposed to be directed towards the centre of vision.

Let us thoroughly understand the relation that exists between the direction of sight and the picture plane, as it illustrates the fundamental law of all vanishing points.

17. We have already shown [13] that any series of receding parallel lines are represented in perspective by lines drawn to a vanishing point.

And we have shown [13] that this vanishing point is where the line from the spectator's eye, parallel to those lines (to which the name of vanishing parallel has been given), meets the picture plane.

We have also stated [16] that the picture plane is at right angles to the direction of sight; consequently, the direction of sight is the vanishing parallel for all lines at right angles to the picture plane. As the direction of sight meets the picture plane in the centre of vision, a point opposite the spectator's eye [15], *all lines at right angles to the picture plane converge towards a point opposite the spectator's eye.* In other words, all lines receding in the same direction as that in which the spectator is looking, appear to converge towards the point the spectator is looking at.

18. THE HORIZON LINE.—Out at sea, on a clear day, a distinct boundary line appears, where the sky and sea appear to meet, called the *horizon.* This line is where the surface of a large sphere—the Earth—is seen against the sky. If the surface of the Earth were an infinite plane, instead of a sphere, there would be no appreciable difference in the position of this boundary line, except from a great height. Its perspective representation will evidently be a horizontal line, and since one point on it is in the "direction of sight," of which C has been shown to be the perspective, it follows that the horizon will be represented in the picture by a horizontal line through C. We call this the *horizon line.*

19. *N.B.*—In every picture the space between the ground line and the horizon line represents in perspective a horizontal plane, viz. the ground plane [2]; infinite in extent towards

the horizon line, but bounded towards the spectator by the ground line [4]. Let this be most carefully realised and remembered.

20. *All receding horizontal lines have their vanishing points on the horizon line,* for it is evident that the distance of each from the ground plane, as seen in perspective, gradually diminishes until it vanishes in the infinite distance, *i.e.* the horizon.

21. Lines Parallel to the Picture Plane.—We have

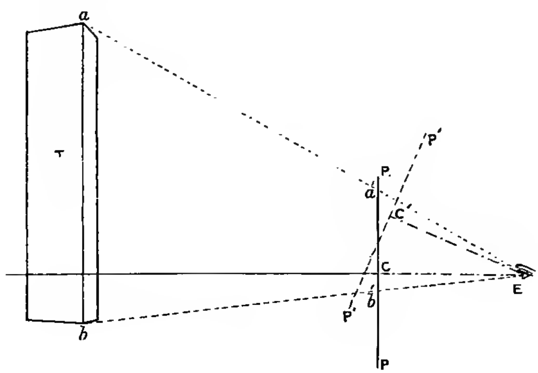


Fig. 22

now to consider lines that are not receding, *i.e.* lines that are parallel to the picture plane.

The edges, *b*, *c*, *d*, of the box (Fig. 20) are lines of this character; and we shall find, by testing the picture traced on the glass, that *with regard to all lines which are parallel to the picture plane, their relative positions and proportions remain unchanged in the picture.*

To guard against a common misconception, it will be as well to consider the matter more closely. With regard to vertical lines, people are apt to argue as follows—"The top of a high tower must be farther from the eye than the bottom, consequently, it must appear smaller." Let T (Fig. 22) represent a high tower, E the position of the eye, P.P. the edge of the

picture plane, and C the centre of vision. Now it is quite true that the point a is farther from E than b, but its representation (a') on the picture plane is also farther off than the representation b' of b, and in the same proportion. Remember the picture plane is vertical and the eye is looking directly at C.

If the spectator were looking towards the top of the tower, in the direction of C′, then the picture plane would have to be moved into the position P.′P.′, as it must always be at right angles to the direction of sight [16], in which case the tower would be no longer parallel to P.′P.′, but its vertical edges would be receding lines and converging towards a vanishing point.

We will take another illustration, as these misconceptions are great stumbling blocks to the proper appreciation of perspective.

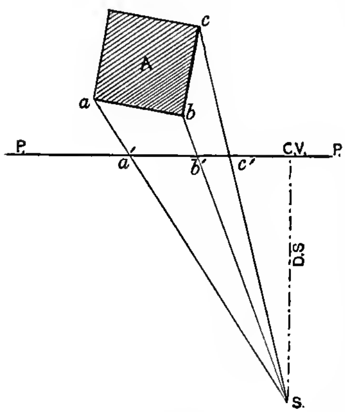


Fig. 23

Fig. 23 is a plan, in which A represents the same tower, S the station point or position of spectator, and P.P. the position of the picture plane: a, b, c represent in plan both the visible top as well as the corresponding bottom corners of the tower. Then the visual rays both from the top and bottom corners are represented in plan by the same lines passing through a', b', c', consequently, a', b', c' are representations in plan of vertical lines on the picture plane, *i.e.* the edges of the tower in the picture are vertical.

22. Surfaces Parallel to the Picture Plane.—It follows from the preceding article that surfaces parallel to the picture plane retain, in the picture, their geometrical proportions; hence in Fig. 20 the corners are correctly drawn as right angles,

and the proportion between the height and width of each box is made the same as in the model.

23. Points of Distance.—We will now take a square piece of paper, with a side of about six inches, having diagonals drawn across it. Place it on the ground plane to the left of the spectator, with one of its edges parallel to the plate glass, or picture plane, and trace it. Its tracing will appear as shown in Fig. 24.

If we now produce the diagonals till they meet the horizon

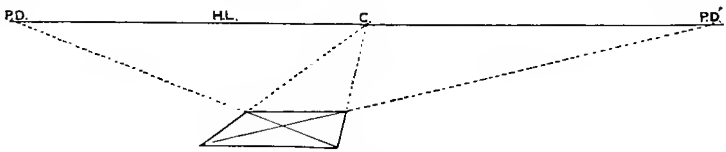


Fig. 24

line, they will give us points P.D. and P.D.'. It is found by measurement that each of these points is equally distant from C, and also that the distance of P.D. and P.D.' from C is equal to the distance of the station point (S) from C; for this reason they are called the *points of distance.*

They are very important points, for they are used in measuring receding distances, or distances from the picture plane.

We proceed to explain their use, and to show why it is that the point C is equidistant from them, and from the eye of the spectator, as above stated. Let A and B (Fig. 25) represent the plan of two cubes, and S the station point; we need not trouble about scale for the present.

Draw the line P.P. for the position of the picture plane, touching the sides of the cubes nearest the spectator, and draw a line D.S. from S, perpendicular to it, for the direction of sight. Where this line meets the picture plane is the centre of vision (C). With this point as centre, and CS as radius, mark off the points P.D. and P.D.' on the line P.P. These are the

points of distance. Join S to P.D. and P.D.′, and draw diagonals to the tops of the cubes, as shown.

Then CS is equal to the distances of P.D. and P.D.′ from C. The angles which CS makes with the lines joining S with

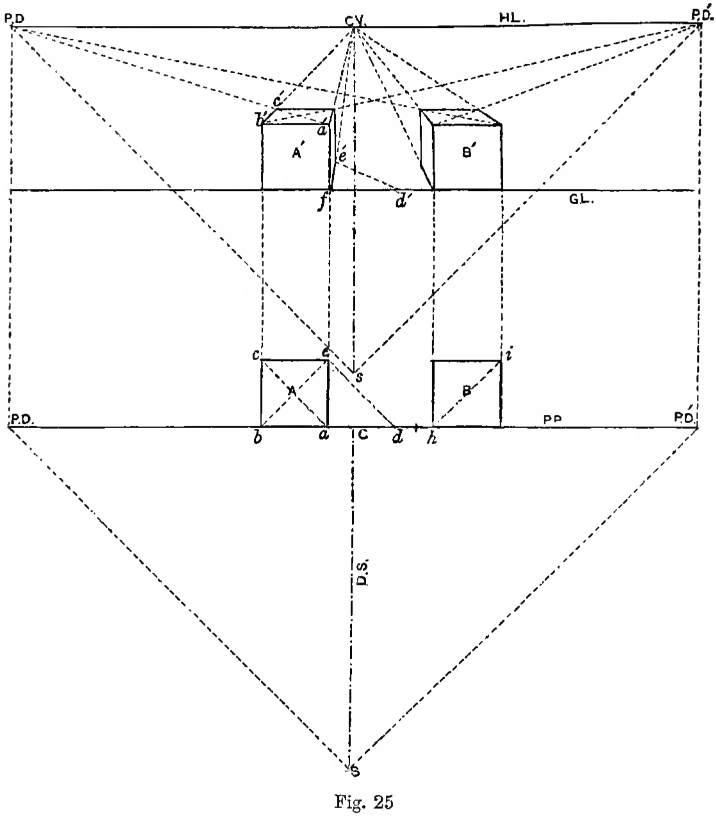


Fig. 25

P.D. and P.D.′ are 45°. Therefore the lines joining S with P.D. and P.D.′ are parallel to the respective diagonals of the plans of the cubes, and consequently are *vanishing parallels* for all lines parallel to them [13].

We will now draw an elevation of the picture plane.

Draw any horizontal line G.L. for the ground line, and also

draw the horizon line H.L. to represent the level of the eye. Carry up from plan the points C, P.D., and P.D.′ Make elevations of the fronts of the cubes, as A′ and B′, by carrying up from the plan perpendicular lines called *projectors.*

As the fronts of these cubes are parallel to the picture plane, the receding edges must be at right augles to it, consequently, they must be drawn to C.V. (the centre of vision) [14].

If we draw a line from *a′* to P.D. it will meet the line drawn from *b′* to C.V. in *c′*; and since the angle *a′b′c′* is the perspective representation of the angle *abc,* which is a right angle, and the angle *b′a′c′* represents the angle *bac* which is 45,° therefore, *b′c′* must be the perspective representation of *bc,* which is equal to *ab,* as will be seen in plan.

As lines parallel to the picture plane are drawn geometrically parallel to each other [21], draw the back of the cube parallel to the line *b′a′.*

The length of the receding edges could also be measured by setting off *fd′* along the ground line, equal to *ae,* and drawing a line from *d′* to P.D., cutting the line from *f* to C.V. in *e′*, and then drawing lines parallel to the front of the cube as before. They could also be measured by drawing a line from *b′* to P.D.′, the proof of which would be exactly similar to that given above.

The student will now see the use of the points of distance, and why they are called measuring points; he will also see that it is not necessary to use both points for this purpose, as either of them will serve for all that is required.

Particular attention should be paid to the following:—We have now shown that all lines parallel to the direction of sight, as *bc, ae,* etc., are drawn to the centre of vision; that all lines parallel to the line drawn from S to P.D., as *ac,* and *de,* are drawn to P.D. on the horizon line; and that all lines parallel to the line drawn from S to P.D.′, as *be,* and *hi,* are drawn to P.D.′ The lines from S to C, P.D., and P.D.′, are all special instances of vanishing parallels [13].

To make the explanation clearer, we drew the plan of the cubes, etc., but this was not really necessary, as we could have combined both plan and elevation in one drawing. For instance, we could have set off the station point S on a perpendicular to the horizon line drawn from C.V. as *s*.

It is usual to divide Linear perspective into three kinds, viz. *Parallel*, *Angular*, and *Oblique*.

24. Parallel Perspective.—A rectangular object is said to be in *parallel perspective* when one face of it is parallel to the picture plane. In such a position it has only one set of receding edges ; consequently, the speciality of parallel perspective is only one vanishing point, as shown in Fig. 25.

25. Angular Perspective.—A rectangular object is said to be in *angular perspective* when its base is on, or parallel to the ground plane, while it has no face parallel to the picture plane, as Fig. 21.

Let A (Fig. 26) represent the plan of a cube, and S the station point.

If the spectator is looking in the direction of C.V. the picture plane must be in the position shown by the line P.P., *i.e.* at right angles to the direction of sight, and the object A will be in *parallel perspective* [24], but if the spectator were to look in the direction of C.V′., the picture plane would be in the position shown by the line P.′P.′, and the object A would be in *angular perspective* [25], because its face would be inclined to the picture plane. It would then have two sets of receding lines, and, consequently, there would be two vanishing points.

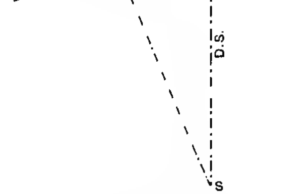


Fig. 26

26. Oblique Perspective.—A rectangular object is said to

be in *oblique perspective* when it has no face parallel to either the picture plane or the ground plane.

N.B.—This branch of the subject belongs to the advanced stage.

We will now draw an object in angular perspective.

Let A (Fig. 27) represent the plan of a rectangular solid with its longer edges receding to the right at an angle of 30°, and let its height be equal to one-third of the edge *ac*. Let S be the station point, and P.P. the position of the picture plane.

27. To find a Vanishing Point.—*To determine the position of the vanishing point for any given receding line, draw a line parallel to it from the eye till it meets the picture plane.*

Let *ab* be the given line (called the *original line*); draw a line from S (the position of the eye) parallel to it to meet the picture plane in V.P., this is the vanishing point required [13]. The line drawn from S, parallel to the given line, is called the *vanishing parallel* of the original line. Determine V.P.′ the vanishing point for *ac* in the same way.

Draw any horizontal line G.L. for the ground line, and assume it to be on the level of the ground plane. Draw the horizon line H.L. to represent a line level with the eye. Draw a projector from *a* to determine the nearest edge of the solid and make *a′e* equal in length to one-third of *ac*. Transfer V.P. and V.P.′ to the horizon line, and draw lines to them from *a′* and *e*, as shown.

To enable us to determine the lengths of the receding sides of the solid we must obtain points called *measuring points.*

28. Measuring Points.—Every vanishing point has a corresponding measuring point.

To find a measuring point for any receding horizontal line, take the distance of the eye from its corresponding vanishing point, and set it off on the horizon line.

We will first determine the position of the measuring points on plan, and then transfer them to the horizon line.

With V.P. as centre, and the distance of S from it as radius, describe an arc to meet P.P. in M.P.: this is the measuring

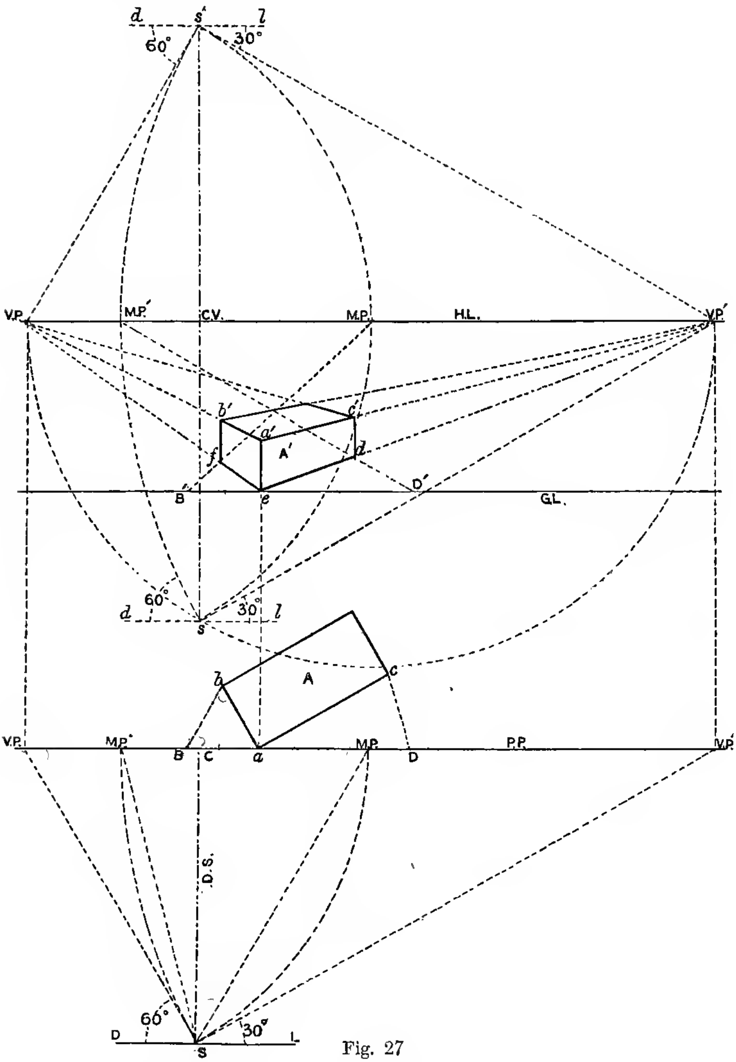


Fig. 27

point corresponding to V.P. Determine M.P.' for V.P.' in a similar manner.

Transfer both these points to the horizon line.

The true length of each edge of the solid receding to the left is *ab*. Set this distance off from *e* on the ground line, as *e*B′, and draw a line from B′ to M.P. to meet the line from *e* to V.P. in *f*. Then *ef* is the perspective length of *ab*. Draw a vertical line at *f* to meet the receding line from *a′* to V.P. in *b′* : *a′b′* is the upper edge of the solid, and this completes its left-hand side. Determine the right-hand side in a similar manner, and complete the solid by drawing lines to the respective vanishing points, as shown.

29. Let us consider these measuring points more closely.

Suppose we look for the vanishing point of the line B*b*. We find it from the principle already laid down [27], by drawing a line from the eye (*i.e.* from S) to meet the picture plane. But since the distance between V.P. and S is equal to the distance between V.P. and M.P., and since, moreover, the line joining V.P. with S is parallel to the line *ab*, and the line joining V.P. with M.P. is parallel to *a*B, it follows that the line drawn from S to M.P. is parallel to B*b*. Therefore M.P. is the vanishing point we are seeking; and it is therefore, we know, the vanishing point for all lines parallel to B*b*, *i.e.* for the bases of all isosceles triangles, of which this angle *a* is the angle of the vertex. This explains why, in order to measure off lengths on receding lines, we first lay off these lengths on the ground line, and then join their extremities to the measuring points, which, as we have shown, are a special class of vanishing points.

The same principle applies to V.P′. and M.P′., the line D′*d* being the base of an isosceles triangle, represented on plan by the line D*c*, which is parallel to a line joining S with M.P.′

This principle is also applicable to parallel perspective, and applying it we shall see that the points of distance are the vanishing points for the bases of such isosceles triangles as we have been speaking of. Therefore the points of distance are

the measuring points corresponding to the centre of vision (see Fig. 25).

30. The points of distance are, as just stated, measuring points corresponding to the centre of vision, but we must beware of taking the centre of vision to be a measuring point for either of the points of distance. If we wish to measure a line drawn to a point of distance, we must find its corresponding measuring point in the same manner as if it were any other vanishing point [28].

31. DIRECTING LINE.—In Fig. 27 the angles of the object with the picture plane are 30° on the right and 60° on the left; knowing this, we could find the vanishing points by setting off these angles at S, with the line D.L. drawn through the station point parallel to the ground line. This line is called the *directing line.*

It is not necessary to draw the plan of an object to put it in perspective; it has been drawn here (Fig. 27) to make the explanation clearer. We could mark off the station point on a vertical line through the centre of vision, either below the horizon line as *s*, or above it as *s'*; the required angles would then be set off from a directing line at either of these points, as shown.

32. It should be particularly noticed that, since the lines joining S with V.P. and V.P.' are at right angles, the two vanishing points, and the station point, lie on a semicircle. This, of course, is always the case for rectangular objects in angular perspective, whatever angles they form with the picture plane.

33. VANISHING LINES.—As receding parallel lines converge to the same point [13], receding parallel planes will converge to the same line. Such a line is called a *vanishing line.*

34. *The vanishing lines for all planes perpendicular to the picture plane must pass through the centre of vision.*

35. *The horizon line is* [20] *the vanishing line for all horizontal planes.*

Let A (Fig. 28) represent a parallel perspective view of the small box we have already used as an illustration in Fig. 20. The face C is a vertical plane perpendicular to the picture plane, so a line drawn through C.V., parallel to the edge *ab*, is the

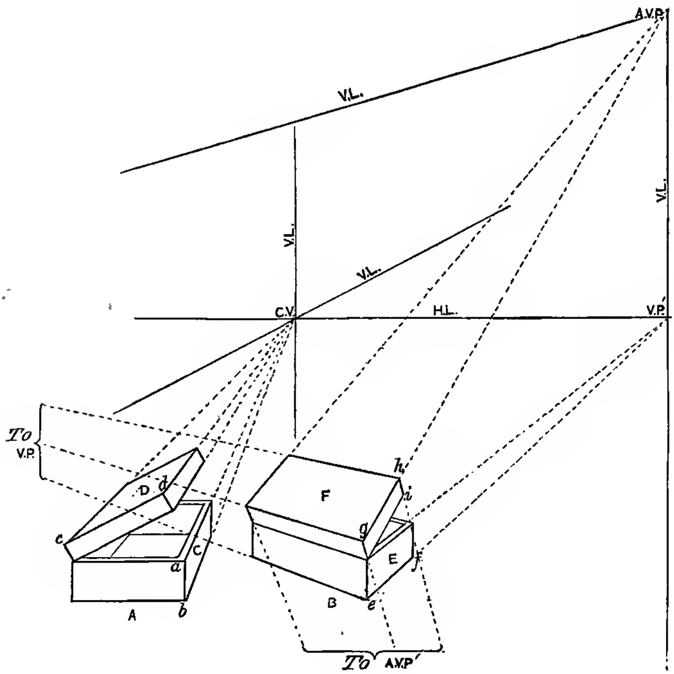


Fig. 28

vanishing line for all planes parallel to C. The face D is also a plane perpendicular to the picture plane, and a line drawn through C.V., parallel to the edge *cd*, is the vanishing line for all planes parallel to D.

36. *The vanishing line for any receding plane must pass through the vanishing point for any horizontal line in that plane.*

This seems scarcely to require proof; but, as an illustration, we may take a particular case. Let B represent the same box in angular perspective; the face E is a receding vertical plane, and V.P.′ is the vanishing point for the horizontal line *ef* in

that plane. A vertical line drawn through V.P.' is the vanishing line for all planes parallel to E. Similarly the vanishing line for the plane F passes through the two points A.V.P. and V.P.

C is a plane perpendicular to the picture plane, and perpendicular to the ground plane.

D is a plane perpendicular to the picture plane, and inclined to the ground plane.

E is a plane inclined to the picture plane, and perpendicular to the ground plane.

37. *Note.*—The plane F is said to be in *oblique perspective*,

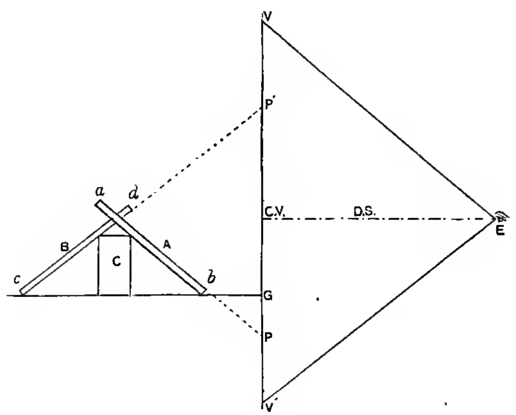


Fig. 29

and is inclined to both the picture and ground planes. The vanishing lines for planes of this character belong to the advanced stage of perspective; in fact, to find any of these vanishing lines from given data belong to that stage, but it is advisable that the student in the elementary stage should understand the general principle of vanishing lines.

38. There are two other kinds of planes to be considered, viz. *ascending* and *descending* planes.

Let A and B (Fig. 29) represent the edges of two rectangular boards resting on a block C, E the position of the eye, D.S.

the direction of sight, and VV′ (a vertical line through C.V.) the edge of the picture plane. The line EV drawn parallel to *ba* gives at V the height of the horizontal vanishing line of the upper and lower surfaces of the board A. The position of the vanishing line for B is found in a similar manner at V′.

We know that the point of distance is at the same distance from C.V. as the distance of the eye from the picture plane, so that, if we know the angles these planes form with the ground

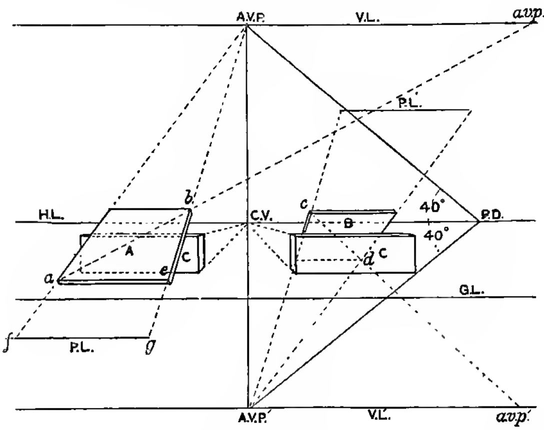


Fig. 30

plane, we can find the position of these vanishing lines without making a separate drawing.

Let each of these planes be inclined to the ground plane at an angle of 40°. Draw lines at this angle, at P.D. (Fig. 30), above and below the horizon line, and produce them till they meet a perpendicular drawn through C.V. Horizontal lines drawn through these points are the vanishing lines required.

39. Accidental Vanishing Points.—We know that receding horizontal lines, parallel to the ground plane, have vanishing points on the horizon line [20], but lines that are inclined to the ground plane converge towards points either above or below the horizon line, called *accidental vanishing points.* A.V.P. (Fig. 30) is the accidental vanishing point for the

edges of the plane A, and any receding line in this plane (as the diagonal *ab*) will have an accidental vanishing point on the vanishing line of this plane as *a.v.p.* The same applies to plane B. A.V.P.′ is the accidental vanishing point of its edges, and a diagonal to the plane will vanish at *a.v.p.*′, a point on the vanishing line of the plane. In Fig. 28 the three lines *ef*, *gh*, and *hi* are all in the same plane (E), consequently they all have their vanishing points in the vanishing line for that plane, which we know is a vertical line through V.P.′

Note.—[37] also applies to accidental vanishing points.

40. Picture Lines.—The intersection of any receding plane with the picture plane gives a line called the *picture line* of that plane. The picture line of the ground plane is called the ground line [4].

41. The plane A (Fig. 29) produced towards the picture plane will meet it in P: this is the position of its picture line. P′ is the position of a similar line for the plane B. These lines are shown in Fig. 30 by the horizontal lines P.L. and P.L.′. The width of the planes A and B could be measured by scale on these lines; for example, *fg* is the true width of the plane A; consequently *picture lines* are also called *measuring lines.*

42. Dividing Points.—It is sometimes necessary to divide a receding line into a given number of equal or unequal parts; this, of course, could be accomplished by means of its measuring point, but it sometimes happens that we have to divide a line that is not parallel to the series of lines for which we have the measuring point. It would be better, in such an instance, to use a *dividing point.*

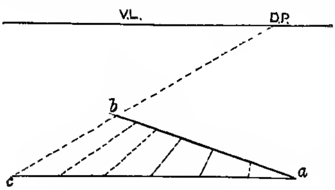


Fig. 31

Let *ab* (Fig. 31) represent a receding line, which we wish to divide perspectively into six equal parts; and let V.L. represent the vanishing line of any plane containing it. If *ab* were a receding horizontal line, the vanishing line of the horizontal

plane containing it would, of course, be the horizon line, but the principle of dividing points applies to receding lines in any position.

From *a* draw *ac* parallel to V.L., and set off from *a*, along it, six equal divisions of any convenient size. Draw a line from *c*, through *b*, to meet V.L. in D.P.; this is the dividing point required. Draw lines from each of the divisions on *ac* to D.P. to meet *ab*, which will then be divided as required.

If we were to set off unequal distances on *ac*, and treat them

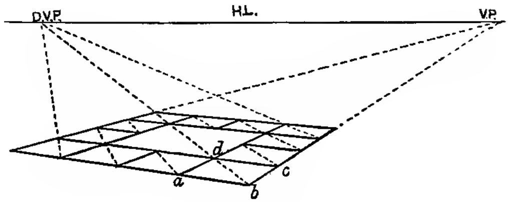


Fig. 32

in a similar manner, then *ab* would be divided, perspectively, in the same proportion as the line *ac*.

Measuring points are sometimes called dividing points because they can be used for this purpose.

43. Diagonal Vanishing Points.—Having given a square drawn in perspective, as *abcd* (Fig. 32), it is sometimes necessary to repeat it, as in drawing a pattern on a floor, etc., this can be more easily accomplished by means of a *diagonal vanishing point*, because it not only simplifies the work, but it answers the purpose of the two measuring points. Draw the diagonal *bd*, and produce it to meet the horizon line in D.V.P.; this is the diagonal vanishing point. We can repeat the original square as often as we choose, by simply drawing diagonals to this point, as shown.

44. The Field of Vision.—It has already been stated [14] that the eye is supposed to be directed towards the centre of vision. The extent of our vision is limited to a certain space

surrounding this point. The *angle of distinct vision* (*i.e.* the angle subtended at the eye) is taken as 60°.

If we assume the position of the eye (Fig. 33) to be the vertex of a right cone, the angle at the vertex being 60°, and the direction of sight (D.S.) its axis, the cone will intersect the picture plane in a circle; the area of this circle is called the *field of vision*. The point on the picture plane, opposite the spectator's eye, is the *centre of the field of vision.* The cone itself is called the *cone of visual rays*. The direction of sight is also called the *principal visual ray.*

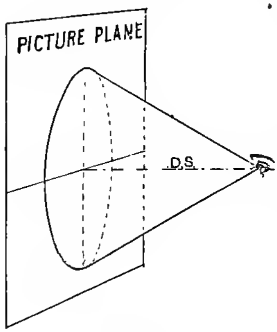


Fig. 33

45. *The field of vision is a space on the picture plane showing the extent of distinct vision. It represents the base of an imaginary cone, subtending an angle of* 60°, *the vertex of which is the position of the eye, and the direction of sight its axis.*

Let S (Fig. 34) represent the station point, A and B the plans of two cubes, C the centre of vision, and P.P. the position of picture plane.

Set off the angle of vision at S in the following manner:—Draw the directing line D.L. at S [31]. With S as centre, and with any radius, describe a semicircle. With the same radius step off the points *e* and *f* on the semicircle, and draw lines from S, through them, to meet the picture plane. This is the cone on plan.

Draw the cubes in perspective, as described for Fig. 25, and draw a projector from *g* to meet the horizon line in *g'*. With C.V. as centre, and the distance of *g'* from it as radius, draw a circle. This is the field of vision.

In practice it is better to determine the radius of the base of the cone by drawing a line from the point of distance, at an angle of 30° with the horizon line, to meet a vertical line through the centre of vision, as shown in Fig. 34.

The cube A′ being inside the field of vision has a natural appearance, but B′ being outside of it appears to have its receding edges too long.

Place another cube D on plan, and draw a visual ray from

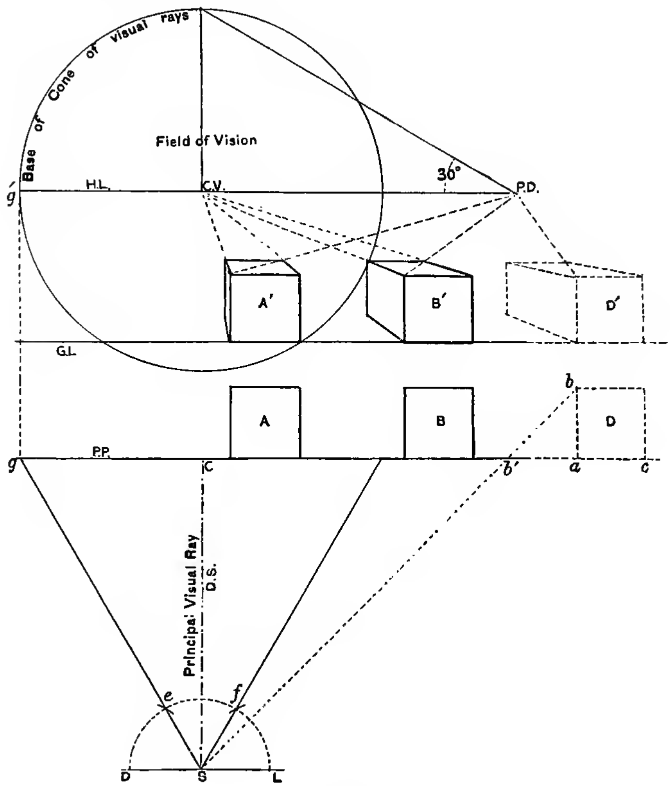


Fig. 34

its corner *b* to S; then *b′a* represents the perspective length of *ab* on the picture plane. Comparing this length with *ac*, the front of the cube, we shall find the receding side appears longer than the front, thereby showing the distortion caused by the visual rays passing too obliquely through the picture plane. D′ is the perspective view of D.

The centre of vision is not necessarily the centre of the

picture; we can use any part of the field of vision to represent the extent of our subject; for instance, the rectangle marked A (Fig. 35) shows the part occupied by the picture by Alma Tadema, R.A., called "The Kiss"; in this subject the centre of vision is on the extreme limit of the drawing, and quite near the top. Our picture could, if desired, occupy the position marked by the rectangle B.

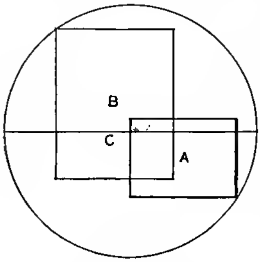


Fig. 35

When the subject is in parallel perspective, especially if it is an interior view, it is considered to have a more pleasing effect when the centre of vision is not in the centre of the picture.

The largest drawing possible of any subject would be circular in shape, and would occupy the whole of the field of vision.

The limit of distinct vision only applies to the object; the vanishing points, etc., may be at any distance from the centre of vision.

46. LINE OF HEIGHTS AND VERTICAL MEASURING POINTS. —It has been stated [7] that objects appear to get smaller as they recede from the eye. Most of the objects we have already illustrated have been drawn touching the ground line, *i.e.* with their nearest parts in contact with the picture plane.

47. The three cubes (Fig. 36) are at some distance from the picture plane, and are said to be *within* the picture. If we were to place their nearest edges *ab* at AB they would touch the picture. The point *e* is *without* the picture.

48. If they were touching the picture, the heights of their nearest edges could be measured off at once by the specified scale, but if they are within the picture, their heights are generally measured by means of a *line of heights and vertical measuring point.*

Any point on the horizon line may be taken for a vertical measuring point, *e.g.* we have a perpendicular *ab* (Fig. 36) upon

which we wish to measure the heights of three cubes. Assume any point V.M.P. on the horizon line, and draw a line from it, through *a*, to meet the ground line in A. Draw a vertical line AB. Set off upon it the heights of the cubes to the specified scale, and draw a line from each division to V.M.P. to meet the vertical line *ab*; thus we get the perspective heights of the cubes at this distance within the picture. AB is the line of heights, and V.M.P. is the vertical measuring point. If we

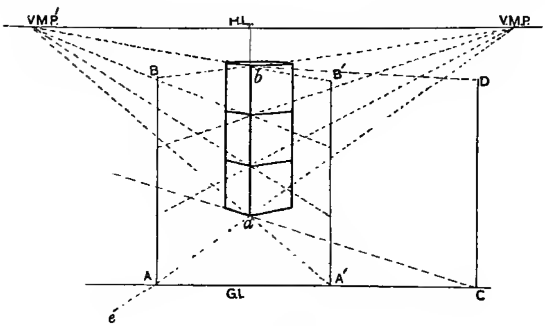


Fig. 36

were to select V.M.P.′ for the vertical measuring point, then A′B′ would be the line of heights.

49. The Point of Contact.—Produce one of the lowest edges of the lowest cube (Fig. 36) to meet the ground line in C; this point is called the *point of contact*, and is, generally, the most convenient position for setting up the line of heights (as CD), because the vanishing point of the object can then be used for the vertical measuring point.

It is immaterial which side of the object is used for this purpose, but the line produced must always be on the ground plane.

Fig. 37 shows the relative positions of the principal points, lines and planes, already described in this chapter.

The line *ab* is receding to the right at an angle of 40°; the line *ac* is receding to the left at an angle of 50°; and the line *de* is at 90° with the picture plane.

These angles are set off at the station point, from the directing line, and *vanishing parallels* are drawn to meet the horizon line, to determine the vanishing points for these angles.

It should be distinctly understood that the station point and direction of sight are not in the same plane with the

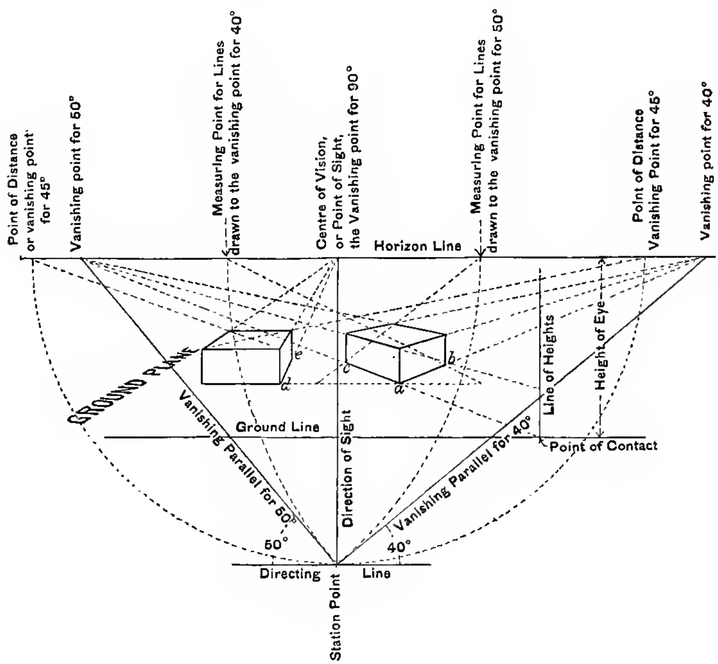


Fig. 37

picture plane, as shown in Fig. 37; the direction of sight is perpendicular to the picture plane, as shown in Fig. 38, which is a perspective view showing their relative position. The vanishing parallel for *ab* (Fig. 38) is parallel to the line *ab*, and determines its V.P.′; in the same way, the vanishing parallel for *ac* is parallel to the line *ac*, and determines its V.P. The direction of sight is the vanishing parallel for the line *de*

(*de* being at right angles with the picture plane), and C.V. is its vanishing point.

It will be seen from this Fig. (38) that it is immaterial whether we draw these vanishing parallels from the point marked station point, or from the point S, both points being on

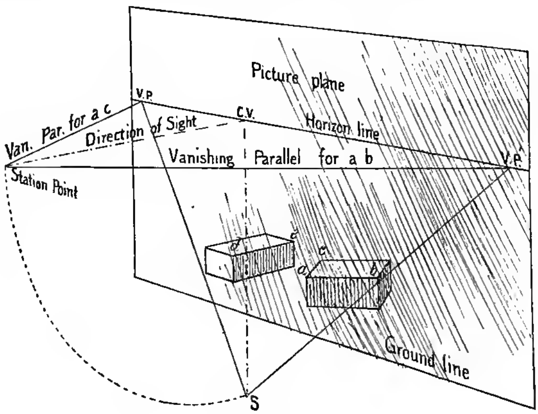


Fig. 38

lines perpendicular to the horizon line, and at the same distance from it.

50. We will now consider the relation between a drawing and the picture plane.

Let us imagine that we have made a perspective drawing of an object, to the *scale of* $\frac{1}{4}$ *full size*—this is a very large scale, but it is convenient for our illustration.

Let A (Fig. 39) represent the object, D the drawing, and E the eye of the spectator; also let *a* be the tracing of the object on the picture plane.

It does not matter how far the object A is within the picture, our drawing is simply a reduced copy to a given scale of the imaginary tracing (*a*) on the picture plane. If we could see the tracing (*a*) on the picture plane, and our drawing were transparent, we should find, on holding it in a position to cover the tracing *a*, that its distance from the eye would be one-

quarter of the distance of the picture plane, *i.e.* *ds* is equal to one-fourth of *ps*.

Where the lower edge of the drawing appears to cut the ground plane is the position of the picture plane; in the illustration it appears to cut it at *p*. If the lower part of the drawing were extended to *e*, the ground line of the picture plane would be at *f*. As its distance from the object would then be increased, the tracing of the object upon it would be smaller, because the visual rays are closer together the nearer they approach the eye; but this will not affect the size of the

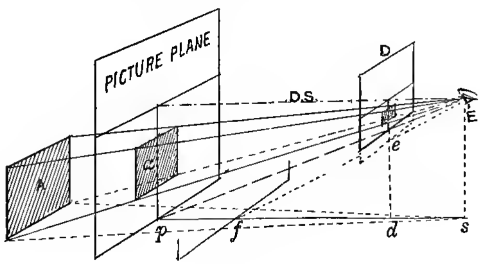


Fig. 39

object on our drawing, it will only alter the proportion between the tracing of the object on the picture plane and its representation on the drawing, *e.g.* *sd* is one-third of *sf*, consequently the object on the drawing will have this proportion to the imaginary tracing on the picture plane, instead of one-fourth, as before. So the amount of foreground we show determines the position of the picture plane.

Having now explained all the principles of the subject, it will be an advantage if we take a sketch from Nature, and show the practical application of some of them.

Fig. 40 is a group of buildings at the entrance of Dartmouth Harbour, comprising the Church of St. Petrox, Dartmouth Castle and fortifications.

As the lines A, B, etc., are geometrically parallel to each other, they must be parallel to the picture plane [21], conse-

quently the line C is at right angles to it; by producing this

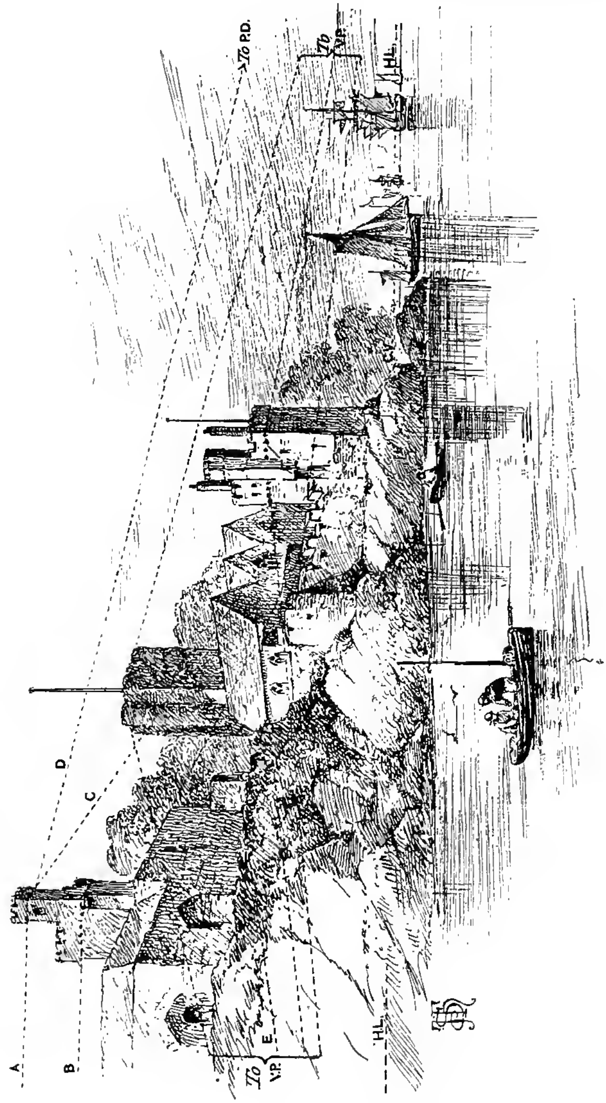


Fig. 40

line to meet the horizon (H.L.), we determine the centre of vision (C.V.).

The line D is the diagonal of the square turret produced. If we were to continue this line to meet the horizon, it would determine the point of distance [23]; so this building is in parallel perspective [24], the other buildings of the group are in angular perspective [25].

The line E is the line produced of the ridge of the nearest roof of the church, and where this line meets the horizon line is the vanishing point for all lines parallel to E. Any of the lines of this building receding to the right could be produced to the horizon to obtain the other vanishing point, but having obtained one vanishing point, with the centre of vision and point of distance, it is possible, from what has been already shown [31] to obtain the other; the method of doing this will be more fully explained in Chapter VIII.

Having obtained these points we could determine the distance from the picture plane, as well as from the spectator, of any part of these buildings, providing its actual size is known; but this will also be explained in Chapter VIII.

Fig. 41 is a practical illustration of the use of accidental vanishing points, etc. If the lines A and B were produced, they would meet in a point which would determine the level of the horizon line (H.L.), for they are receding horizontal lines [20]; this point would be the vanishing point for all lines parallel to A and B. The same rule applies to the lines C and D.

If the line E were produced to the horizon line it would determine the vanishing point for all the horizontal lines of the gate.

The lines F and G produced would determine the accidental vanishing point for all lines parallel to them. The same rule applies to the lines K and L. These two accidental vanishing points would be on a vertical line drawn through the vanishing point for the lines C and D. This line is the vanishing line for all planes parallel to the plane containing the lines G and K (see Fig. 28).

The figure is introduced to show the scale of the sketch.

As the horizon line is below the level of the eyes of the figure, it shows that the eye of the spectator is also at a lower level [18].

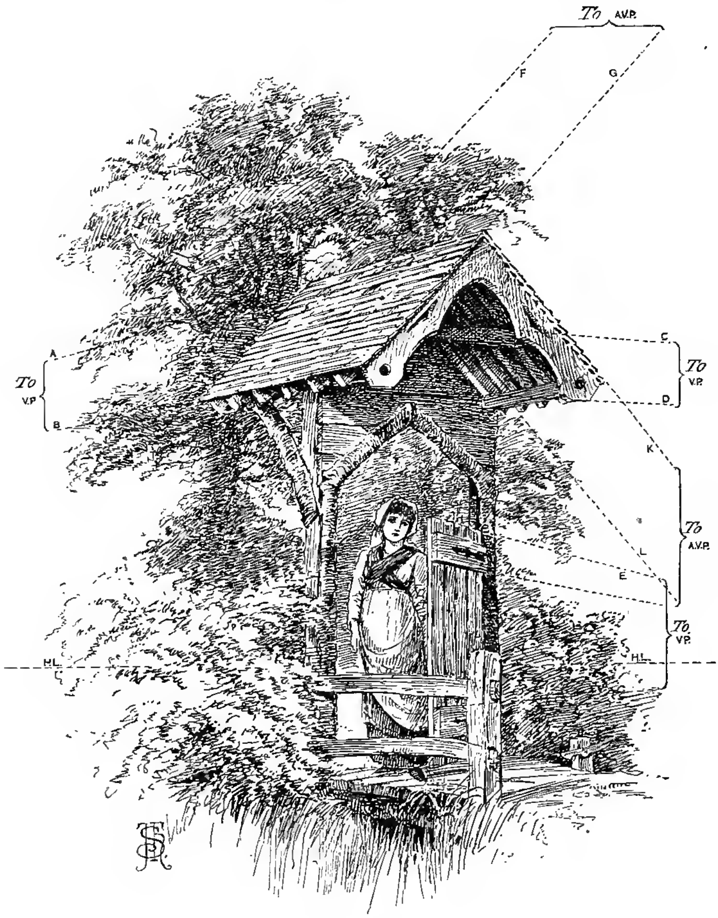


Fig. 41

Candidates for the elementary stage of the Science and Art examinations will be expected to answer some simple questions on the theory of perspective.

The following questions on the theory of perspective, several of which have been adapted from the questions given in Science

and Art examination papers, will test the student's knowledge of the points treated of in this chapter. References to articles containing the necessary information are given in brackets.

EXERCISES

Theory

1. To what line do all receding horizontal lines converge [20]? State what horizontal lines have no vanishing point [21].

2. Define the following terms: Centre of field of vision [14]; line of direction [15]; angle of vision [44]; also state the relations they bear to each other.

3. Define the picture plane [5] and the ground plane [2], and explain their relative positions [3].

4. What is the nearest edge of the ground plane in the picture called [4], and how far does the ground plane extend [19]?

5. Define the line of direction. What important points lie upon it [15]?

6. How are points of distance found, and what lines converge towards them [23]?

7. What lines converge to the centre of vision [14]?

8. Why is the horizon line the vanishing line of all horizontal planes [33]?

9. State the rule for finding the vanishing point for any given receding line [27].

10. How do measuring points enable us to measure receding lines [29]?

11. What is the relation between the ground line and the horizontal line [19]?

12. State, briefly, what you consider the essential principle involved in the perspective representation of an object [9].

13. What is the cone of visual rays [44]? State its usual limits [45].

14. How are measuring points found [28]?

15. How are lines drawn to the points of distance measured [30]?

16. How are lines drawn to the centre of vision measured [23]?

17. Why should the perspective representations of receding parallel lines converge [7]? How can this be proved (Fig. 20)?

18. What is the station point [11]; and what is its relation to the horizontal line [15 and 16]?

19. Define the terms *within* and *without* the picture [47].

20. What is linear perspective [1]?

21. What are visual rays [9]; and what is the principal visual ray called [44]?

22. What is the difference between parallel and angular perspective [24 and 25]?

23. Describe certain parallel lines that have no vanishing point [21].

24. What is a plan? Explain the difference between a plan and an elevation (Fig. 14).

25. Explain the use of the directing line [31].

26. What are picture lines [40]?

27. Where do receding parallel lines meet when they are not horizontal [39]?

28. What is the line of heights [46]?

29. What is the point of contact [49]?

30. What is a dividing point [42]?

31. What is a vertical measuring point [46]?

32. What is a diagonal vanishing point, and what advantage is gained by using it [43]?

33. Describe how the position of the picture plane affects the representation of the object [50].

34. What is the difference between the perspective horizon and the natural horizon [18]?

35. What is a measuring line [41]?

CHAPTER III

PERSPECTIVE FOR SCIENCE AND ART STUDENTS

ELEMENTARY STAGE

Points, Lines, Planes, and Simple Rectangular Solids

Note.—The student should work out the following problems to twice the specified scale.

In the preceding chapter abbreviations have been omitted, in order that by the repetition of the names of the different points, etc., the student might more readily become acquainted with them, but in the succeeding chapters the following abbreviations will be used :—

P.P. = Picture Plane	P.D. = Point of Distance
G.P. = Ground Plane	V.P. = Vanishing Point
G.L. = Ground Line	M.P. = Measuring Point
H.L. = Horizon Line	D.L. = Directing Line
D.S. = Direction of Sight	V.L. = Vanishing Line
C.V. = Centre of Vision	A.V.P. = Accidental Vanishing Point

Note.—The numerals enclosed in brackets refer to the numbers of the articles in Chap. II.

Points in given Positions

Note.—Feet are expressed by one dash (′), and inches by two dashes (″), thus 3 feet 6 inches would be written—3′ 6″.

Fig. 42 represents a cube of 2′ edge in parallel perspective [24].

Distance of P.P. = 9′. *Height of spectator's eye* = 5′. *Scale* $\frac{1}{4}'' = 1'$.

A is a point on the G.P. directly opposite the spectator, and 4′ from the P.P., *i.e.* at a distance within the picture equal to *aa′*.

B is on the G.P., 4′ from the G.L., and 2′ to spectator's left,

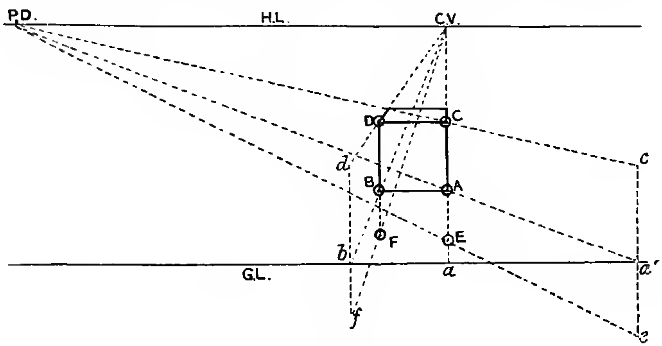


Fig. 42

i.e. at a distance within the picture equal to *aa′*, and to spectator's left equal to *ab*.

C is 4′ from the P.P., and 2′ above the G.P., *i.e.* at a distance within the picture equal to *aa′*, and a height above the G.P. equal to *a′c*.

D is 4′ from the P.P., 2′ to spectator's left, and 2′ above the G.P., *i.e.* at a distance within the picture equal to *aa′*, to spectator's left equal to *ab*, and at a height above the G.P. = *bd*.

E is 4′ from the P.P., and $1\frac{1}{2}'$ below the G.P., *i.e.* at a distance from P.P. = *aa′*, and at a distance below the G.P. = *a′e*.

F is 4′ from the P.P., 2′ to the spectator's left, and 1′ below the G.P., *i.e.* at a distance from the P.P. = *aa′*, to the spectator's left = *ab*, and below the G.P. = *bf*.

We will now proceed to find each of these points in perspective from the given data.

PROBLEM 1

Determine the positions of the following points in perspective. Distance of P.P. $= 9'$. *Height of spectator's eye* $= 5'$. *Scale* $\frac{1}{4}'' = 1'$. Fig. 43.

A *to be on the* G.P., *and in the* P.P. $3'$ *to spectator's right.*

B *to be on the* G.P., $2'$ *from* P.P., *and directly opposite the spectator.*

C *to be on the* G.P., $6'$ *from* P.P., *and* $5'$ *to spectator's left.*

D *to be opposite the spectator,* $4'$ *from the* P.P., *and* $3'$ *above the* G.P.

Draw G.L. to represent the level of the G.P. [4], and $5'$ above it at the level of the eye, draw H.L. [18].

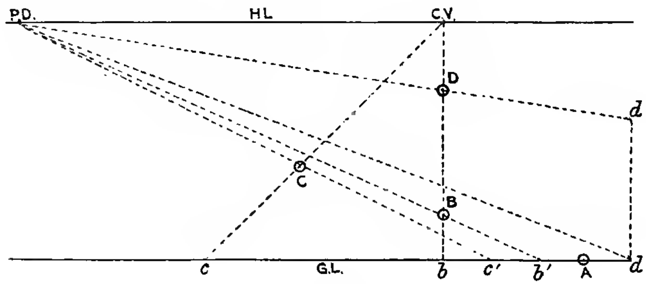


Fig. 43

Assume the point C.V. to be a point opposite the eye [14], and set off $9'$ from it for the P.D. [23].

Draw a perpendicular from C.V., meeting the G.L. in b. Set off $3'$ from b on the G.L. to the right. This is the point A.

To find the point B, set off bb' on the G.L. ($2'$), and draw b' to P.D., meeting a perpendicular on b in B, the point required.

Set off bc $5'$ to the left, and join c to C.V. Set off $6'$ to the right of c, as c', and draw a line from c' to P.D., cutting the receding line from c in C. C is the point required.

Set off $bd = 4'$. Draw the perpendicular $dd' = 3'$. Join d' to P.D., meeting the perpendicular on b in D. D is the point required.

Problem 2

Determine the positions of the following points.
Distance of P.P. = 9′. *Height of spectator's eye* = 5′. *Scale* $\frac{1}{4}'' = 1'$. Fig. 44.
E *to be* 1′ *from* P.P., 2′ *to spectator's right, and* 6′ *above* G.P.
F *to be opposite the spectator,* 4′ *from* P.P. *and* 2′ *below* G.P.
G *to be* 5′ *from* P.P., 6′ *to spectator's left, and* 1′ *below* G.P.

Draw the G.L. and H.L., and mark the points C.V. and

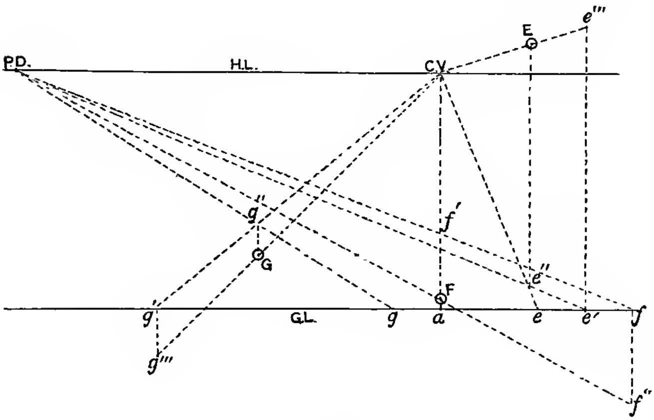


Fig. 44

P.D. as in the preceding problem. Draw a perpendicular from C.V. to meet G.L. in a. Set off ae (2′), and join e to C.V. Set off ee' (1′), and draw e' to P.D., intersecting the receding line from e in e''. Erect a perpendicular at e', 6′ high, which gives e'''. Join this point to C.V., cutting a perpendicular on e'' in E, the point required.

Set off af (4′). Draw f to P.D., cutting the receding line from a in f'. Draw a perpendicular from f 2′ long, which gives f''. Join this point to P.D. to meet a perpendicular from f' in F, the point required.

Set off $ag' = 6'$, and draw g' to C.V. Set off $g'g = 5'$, and draw g to P.D., cutting the receding line from g' in g''. Draw

the perpendicular $g'g''' = 1'$, and draw g''' to C.V., cutting a perpendicular from g'' in G, the point required.

The following is an inverse problem, *i.e.* points are given in perspective, and their positions are to be determined.

It will be seen from the preceding problems that the position of a point is determined by the three following distances, viz.—(1) Its distance from the P.P. (2) Its *lateral* distance, *i.e.* its distance to the right or left of the centre of the picture plane. (3) Its *vertical* distance above or below the G.P. And if a point is given in perspective, and one of these distances is known, the others can be determined.

Problem 3

Determine the positions of the following points given in perspective, with reference to the spectator, P.P., and G.P.
Distance of P.P. $= 9'$. *Height of eye* $= 5'$. *Scale* $\frac{1}{4}'' = 1'$. Fig. 45.

1. A *is on the* P.P. 2. B *is on the* G.P. 3. C *is 2′ to spectator's left.* 4. D *is 6′ from* P.P. 5. E *is* $1\frac{1}{2}'$ *above the* G.P.

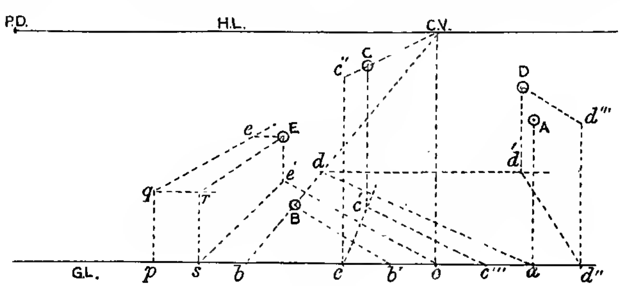


Fig. 45

1. When a point is in the P.P., its position can be determined at once by the specified scale, *e.g.* A will be found to be 2′ to spectator's right, and 3′ above the G.P.

2. Draw a line from C.V., through B, to meet the G.L. in *b*. Draw a line from P.D., through B, to meet the G.L. in *b′*.

Then *bo* is its distance to spectator's left, and *bb'* its distance from P.P.

3. Set off $oc = 2'$, and draw *c* to C.V. Draw a perpendicular from C to meet this line in *c'*. Draw a line from C.V., through C, to meet a perpendicular on *c* in *c''*. Draw a line from P.D., through *c'*, to meet the G.L. in *c'''*. Then *co* is its distance to spectator's left, *cc'''* its distance from P.P., and *cc''* its height above the G.P.

4. Set off on the G.L., from any line drawn to C.V., the point *b* for instance, $ba = 6'$, and draw *a* to P.D. to meet the receding line from *b* in *d*. Draw a horizontal line from *d* to meet a perpendicular from D in *d'*. Draw a line from C.V., through *d'*, to meet the G.L. in *d''*. Draw a line from C.V., through D, to meet a perpendicular on *d''* in *d'''*. Then *od''* is its distance to spectator's right, *ba* its distance from P.P., and *d''d'''* its height above the G.P.

5. Draw any horizontal line *qr*, $1\frac{1}{2}'$ above the G.L., and draw a line from C.V., through E, to meet this line in *r*. Draw a perpendicular from *r* to meet the G.L. in *s*. Draw a line from *s* to C.V. to meet a perpendicular from E in *e'*. Draw a line from P.D., through *e'*, to meet the G.L. in *o*. This happens to coincide with *o*, the foot of the perpendicular from C.V. on the G.L. Therefore *os* is its distance to spectator's left; *os* is also its distance from P.P., and *sr* its height above the G.P.

PROBLEM 4

Draw, in parallel perspective, 9 *cubes of* 1' *edge, all standing on the ground plane,* 1' *apart, symmetrically arranged in* 3 *rows forming a square. The front faces of the front row all lie in the picture plane, and the right-hand edge of the middle cube, of this row, is directly opposite the spectator.*

Distance of P.P. $= 8'\ 6''$. *Height of spectator's eye* $= 4'$. *Scale* $\frac{1}{3}'' = 1'$. Fig. 46.

Draw G.L. to represent the level of the G.P.; and 4' above

it (the height of spectator's eye) draw H.L. Assume C.V. to be a point opposite the spectator's eye, and set off from it, along H.L., 8′ 6″ for the P.D.

Draw a perpendicular from C.V. to meet G.L. in *a*. Draw the front face of the middle cube of the front row touching this line, as shown, and at a distance of 1′ on each side of it, draw another equal square.

Draw the receding lines of the three cubes to C.V.

To determine the tops and spaces between the cubes, draw a line from the corner *b*, of cube A, to P.D. to meet the receding

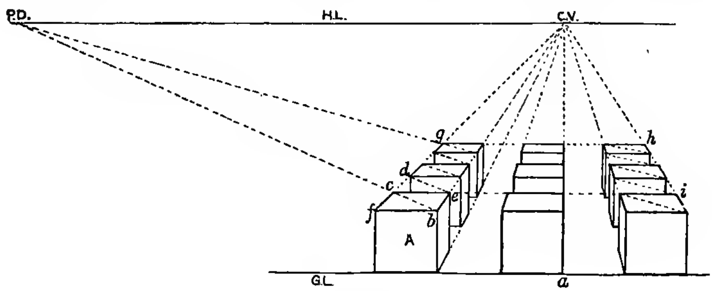


Fig. 46

line from corner *f* in *c*. Draw the horizontal line *ce* parallel to *fb*; then draw a line from *e* to P.D. to meet the receding line from *f* in *d*, and so on till the five receding feet are determined. If we were to treat each row of cubes in the same manner we should find that lines joining the corresponding cubes in each row formed parallel lines, as *ci* and *gh*, as evidently they ought, since the corresponding points of each row are at the same distance from P.P.

Having obtained all the tops of the cubes, draw perpendicular lines for the sides, and complete them as shown.

Perspective Scales

When a number of points have to be found in perspective, the work can be greatly facilitated by means of two receding

parallel lines, one foot apart, drawn to the C.V., called a *perspective scale.*

This scale enables us to determine the relative size of a foot at any distance from the P.P. Any two lines, one foot apart, drawn to the C.V., would answer this purpose, *e.g.* any two of the receding lines forming the top edges of the cubes in Fig. 46, but it is more convenient to place them as shown by the lines drawn from *a* and *b* to C.V. in Fig. 47.

PROBLEM 5

Draw the following lines in perspective by the positions of their extremities, using a perspective scale.

Distance of P.P. $= 8'$. *Height of spectator's eye* $= 4'$. *Scale* $\frac{1}{3}'' = 1'$. Fig. 47.

A *to have both ends* $1'$ *from* P.P.; *one end to be* $1'$, *and the other* $4'$ *to spectator's left.*

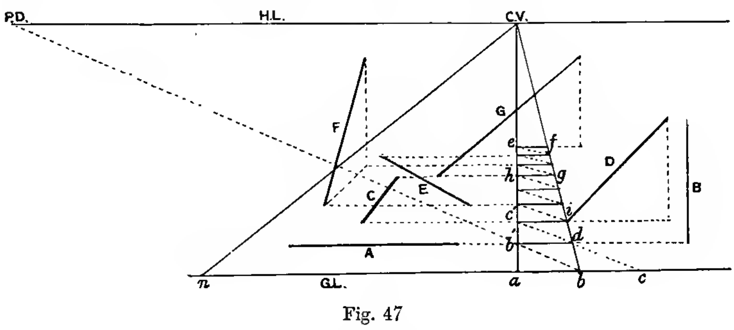


Fig. 47

B *to have both ends* $1'$ *from* P.P. *and* $3'$ *to spectator's right; one end to rest on the* G.P., *and the other* $2'$ *above it.*

C *to rest on the* G.P., *with both ends* $3'$ *to spectator's left; one end to be* $2'$, *and the other* $5'$ *from* P.P.

D *to have both ends* $2'$ *from* P.P.; *one to rest on the* G.P., $1'$ *to spectator's right, and the other* $3'$ *to his right and* $2'$ *above the* G.P.

E *to lie on the* G.P., *with one end* 3′ *from* P.P. *and* 1′ *to spectator's left; the other end to be* 7′ *from* P.P. *and* 4′ *to his left.*

F *to have both ends* 4′ *to spectator's left; one to be* 3′ *from* P.P. *and resting on the* G.P., *the other end to be* 6′ *from* P.P. *and* 3′ *above the* G.P.

G *to have one end resting on the* G.P., 5′ *from* P.P. *and* 2′ *to spectator's left; the other end to be* 8′ *from* P.P., 2′ *to his right, and* 3′ *above the* G.P.

Draw the G.L. and 4′ above it (the height of the eye) draw the H.L. Assume C.V. to be a point opposite the spectator's eye, and set off 8′ (the distance of P.P.) from C.V. for the P.D.

Draw a perpendicular from C.V. to meet the H.L. in a. Set off $ab = 1'$ and join b to C.V. This forms a perspective scale. Draw a line from b to P.D., cutting the perpendicular at a in b'. Draw the line $b'd$ parallel to H.L. and produce it to the left. Set off the distances of the points forming the extremities of line A on this line, using the *perspective foot* $b'd$ for this purpose.

Produce $b'd$ to the right, and from b' set off three perspective feet equal to $b'd$. This will give the lower end of line B. Erect a perpendicular from this point twice $b'd$ in length. This is the line required.

Measure another receding foot in the perspective scale by drawing d to P.D. to meet the receding line from a in c'. Draw the horizontal line $c'i$, and produce it to the left. Set off from c' three perspective feet each equal to $c'i$. This will determine the nearer end of line C. Measure three more receding feet in the perspective scale to obtain the point h. Draw a horizontal line through this point to the left, and set off three perspective feet equal to hg from h. This will determine the farther end of line C. Join these points. This line produced should vanish in C.V.

Determine all the remaining points in the same manner, and join them to obtain the lines D, E, F, and G.

If we set off 5′ from a, as an, and draw n to C.V., we get a

perspective scale for 5′, then $na : ab : : 5 : 1$, and any lines drawn parallel to G.L., throughout these two scales, have this same proportion. If we wish to measure a long distance from the P.P. it would be better to use the larger scale and get five receding feet at a time instead of one foot as in the smaller scale, or we could construct a perspective scale for ten feet and get ten receding feet at a time.

Each of the lines illustrated in Fig. 47 has special characteristics, *e.g.*—

A is parallel to the P.P. and parallel to the G.P.
B " , " perpendicular to the G.P.
C is perpendicular to the P.P. and parallel to the G.P.
D is parallel to the P.P. and inclined to the G.P.
E is inclined " " parallel " "
F " " " inclined " " but in a plane perpendicular to both the P.P. and G.P.
G. is inclined to both the P.P. and G.P.

We will first take lines similar to A, B, and C, in position, and draw them from given data; then we will take planes constructed from such lines, after which we will determine rectangular solids formed by such planes.

If we assume the perpendicular at a to be the edge of the P.P., and P.D. the station point, a visual ray [9] drawn from b to P.D. will give the point b' on the P.P., consequently ab' will represent its receding distance on the P.P.; and as ab is a foot, ab' must represent a receding foot. If we were to draw a line from c to P.D. (2′ from a), ac' would represent its receding distance on the P.P. It will be seen that a line drawn from d would answer the same purpose as drawing it from c; so every succeeding line in the perspective scale drawn to P.D. will measure an additional receding foot from the P.P.

A point e halfway between the G.L. and the H.L. will always represent a receding distance equal to the distance of the station point from the P.P. Since e is 8′ from the P.P., the same distance as P.D. is from C.V., if we were to draw a line

from P.D. through *e* it would meet the G.L. in a point the same distance from *a* as P.D. is from C.V. The line *ef* being twice as far from the spectator as *ab* will be half as long [7].

The line drawn from *a* to C.V. being a receding line lying on the G.P., its distance below the H.L. will always be the same at any distance from the P.P. If we take any horizontal line in the perspective scale, called a *perspective foot,* we shall find that it is four times its own length below the H.L., because the height of the eye is 4′.

The use of the perspective scale enables us to dispense with the line of heights [46], as by its use we can determine the apparent size of a foot at any distance from the picture. It is therefore unnecessary to produce every line to meet the P.P. in order to measure it. This is a great advantage, and when we come to measure receding lines and planes inclined to the G.P., we shall find our work greatly facilitated by the use of the perspective scale.

If we wished to find a point at a long distance from P.P. it would be advisable to construct a perspective scale for 10 feet or more, according to the distance required; we will illustrate this in the following problem.

Problem 6

Find the position of a point A, 124′ *from* P.P., 80′ *to spectator's left, and* 20′ *above the* G.P.

Distance of P.P. = 12′. *Height of eye* = 5′. *Scale* $\frac{1}{4}'' = 1'$. Fig. 48.

Determine the G.L., H.L., C.V., and perspective scale for one foot (Prob. 5).

Set off $ac = 10'$, and draw *c* to C.V. It will be better to measure off the odd number of feet first, so set off $cd = 4'$ and draw *d* to P.D., meeting the receding line from *c* in *e*. Draw a horizontal line through *e*, as *ef*, and draw *f* to P.D. to obtain *g*. As *ef* is 10′ long, *g* must be 14′ from P.P. Draw a horizontal

line through *g*, as *gh*, and produce it to *i*, making *gi* = *gh*. Draw a line from *i* to C.V., and draw a line from *h* to P.D. to meet this line in *j*. As *ih* = 20′, *j* must be 34′ from P.P. Draw a horizontal line through *j*, and make *jm* = *kl*. Draw *m* to C.V., and *l* to P.D., to meet this line in *n*. As *ml* = 30′, *n* must be 64′ from P.D. Draw a horizontal line through *n*, and make *np* = *no*.

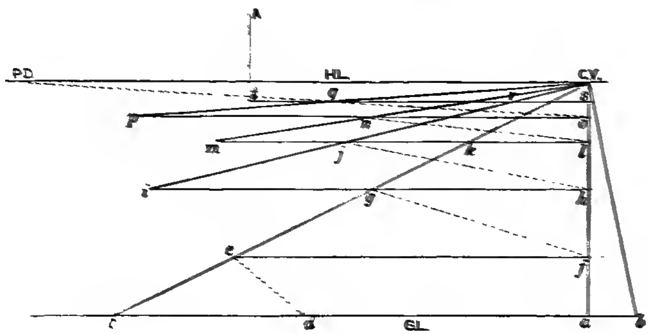


Fig. 48

Draw *p* to C.V., and *o* to P.D., to intersect each other in *q*. As *po* = 60′, *q* must be 124′ from P.P. This is the distance required.

Draw a horizontal line through *q*, and make *qt* = *rs*. As *q* is 60′ to spectator's left, and *rs* = 20′, *t* must be 80′ to the left of the spectator. This is the distance required.

At *t* erect a perpendicular *t*A = *rs*. A is the point required.

If we had to obtain a point 23′ above the G.P., we could obtain the size of a foot at this distance in the perspective scale *ab*, three of which added to *rs* would give the height of the perpendicular at *t*.

Problem 7

Draw the following lines in perspective.

A *to be* 3′ *long, lying on the* G.P. *parallel to the* P.P., *with its nearer end* 2′ *to spectator's left and* 2′ *from the* P.P.

B *to be* 6′ *long, perpendicular to the* G.P., *with its lower end resting on the* G.P., 6′ *from* P.P. *and* 3′ *to spectator's left.*

C *to be* 5′ *long, lying on the* G.P. *perpendicular to the* P.P., *with its nearer end* 3′ *from* P.P. *and* 6′ *to spectator's left.*

Distance of P.P. = 8′. *Height of eye* = 5′. *Scale* $\frac{1}{4}''$ = 1′. Fig. 49.

Draw G.L. to represent the level of the G.P. Set up 5′ for the height of the eye, and draw H.L. Assume C.V. to be a point opposite the eye, and set off from it P.D. equal to the

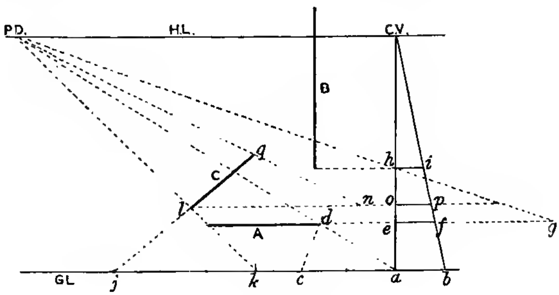


Fig. 49

distance of P.P. Draw a perspective scale for one foot, as *ab* (Prob. 5).

Set off *ac* (2′) and draw *c* towards C.V. to meet a line drawn from *a* to P.D. This gives *d*, the right-hand end of A. To obtain the size of a perspective foot at this distance, draw a horizontal line from *d* through the perspective scale, as *ef*. Measure the length of A with this foot.

We have now to find a distance of 6′ from P.P. for the lower end of B. Produce *ef* to the right, and set off *eg* equal to four times *ef*. Draw *g* to P.D. to meet the receding line from *a* in *h*. Draw a horizontal line through *h*, and set off 3 feet to the left, each equal to *hi*. This is the lower end of B. Erect a perpendicular on this point, six times *hi* in length, to complete the line B.

Set off 6′ to the left of *a*, each equal to *ab*, to find *j*; draw a line from *j* to C.V. Set off *jk* = 3′, and draw *k* to P.D.

to meet the line from j to C.V. in l. Draw a horizontal line from l to obtain the perspective foot op. Set off ln equal to five times op, and draw n to P.D. to meet jl produced in q. Then lq is the line C.

PROBLEM 8

Draw in perspective the three following rectangular planes, each to be $3' \times 2'$.

A *to be perpendicular to both the* P.P. *and* G.P., *with one of its longer edges resting on the* G.P., *and its nearest edge* 1′ *from* P.P. *and* 3′ *to spectator's right.*

B *to lie on the* G.P., *with its longer edges parallel to* P.P., *and its nearest corner* 2′ *from* P.P. *and* 1′ *to spectator's left.*

C to be parallel to P.P., *with one of its longer edges resting on the* G.P., *and its right-hand edge* 5′ *from* P.P. *and* 2′ *to spectator's left.*

Distance of P.P. = 8′. *Height of eye* = 4′. *Scale* $\frac{1}{8}'' = 1'$. Fig. 50.

Determine the G.L., H.L., C.V., P.D., and perspective scale as in Prob. 5.

Find the receding foot ac, and draw ce equal to three times cd.

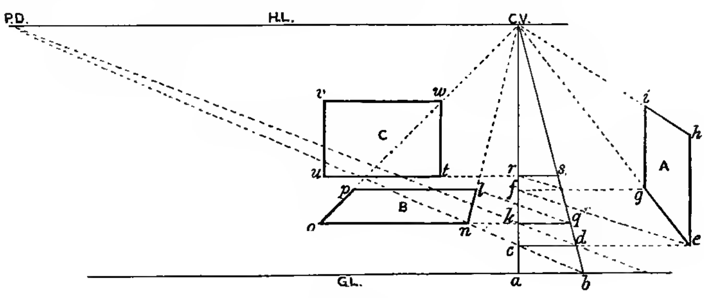


Fig. 50

Draw e to C.V. Erect a perpendicular eh = twice cd, and draw h to C.V. Draw e to P.D. to meet the receding line from a in f. Draw a horizontal line from f to meet the receding line

from e in g. Draw a perpendicular on g to meet the receding line from h in i.

Then $hegi$ is the plane A.

Find the point k, 2′ from P.P., and draw a horizontal line through it. Set off $kn = kq$, and draw n to C.V. Draw no = three times kq, and draw o to C.V. Draw q to P.D. to find l, and draw the horizontal line lp.

Then $nopl$ is the plane B.

Find the point r, 5′ from P.P., and draw a horizontal line through it. Set off rt = twice rs, and tu = three times rs. Erect perpendiculars at t and u, each = twice rs, and join their tops.

Then $tuvw$ is the plane C.

Problem 9

Draw a rectangular solid of dimensions 3′, 2′, *and* 1′ *respectively in parallel perspective, with its longest edges receding, and its shortest edges perpendicular to the* G.P.; *its nearest vertical edge to be* 1′ *from* P.P. *and* 2′ *to spectator's right. Distance of* P.P. = 8′. *Height of eye* = 4′. *Scale* $\frac{1}{3}''$ = 1′. Fig. 51.

Determine the G.L., H.L., C.V., P.D., and perspective scale, as in Prob. 5.

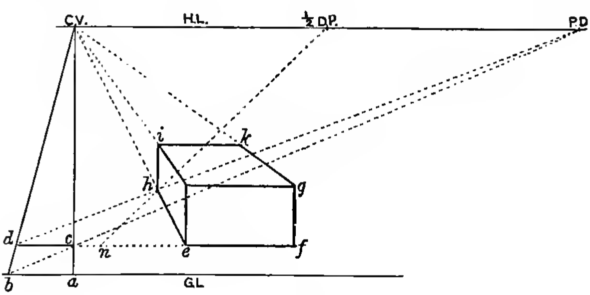


Fig. 51

Draw a line from b to P.D. to meet in c the line joining a with C.V. Draw a horizontal line through c, and set off

ce = twice *cd*. Draw *ef* = twice *cd*, and the perpendiculars on *e* and *f* each = *cd*. Join the tops of these lines to complete the front of the solid. Draw the receding lines to C.V. As these receding lines are to be 3′ long, set off three perspective feet, each = *cd*, from *e*, as *ed*; and draw *d* to P.D. to meet in *h* the receding line from *e*. Draw the perpendicular *hi*, and the horizontal line *ik*, to complete the solid.

Half-Distance Points

When we are drawing objects to a large scale it sometimes happens that we have not sufficient room on our paper to measure off the P.D. When this is the case we can take half the required distance and call it the *half-distance point*. But we must be careful, in measuring the receding line, to set off only half of the required length, *e.g.* in Fig. 51 the point marked ½ D.P. is halfway between C.V. and P.D., and if we set off *en* = half of *ed*, and draw it to the ½ D.P., it will meet the receding line from *e* in the same point (*h*) as the line drawn from *d* to P.D.

As we are restricted by the size of the page in this book, it will be an advantage in most of the problems to use this point in preference to the P.D.

Problem 10

Draw the following solids, in the positions shown, by parallel perspective, using a ½ D.P.

A *to be* 5′ × 5′ × 1′; B *to be* 3′ × 3′ × 1′; *and* C *and* D *cubes of* 2′ *edge. The nearest vertical edge of* A *to be* 2′ *from* P.P. *and* 2′ *to spectator's left.*

Distance of P.P. = 10′. *Height of eye* = 5′. *Scale* ½″ = 1′. Fig. 52.

Determine the G.L., H.L., C.V., and perspective scale, as in

Prob. 5. As the distance of P.P. is 10′, take half this distance from C.V. along H.L. for the $\frac{1}{2}$ D.P.

As $ab = 1'$, a line drawn from b to $\frac{1}{2}$ D.P. will give two receding feet on the receding line from a, as ac.

Draw a horizontal line through c to determine the perspective foot cd. Set off $ce = 2$, and $ef = 5$ perspective feet. Make $eg = 1$

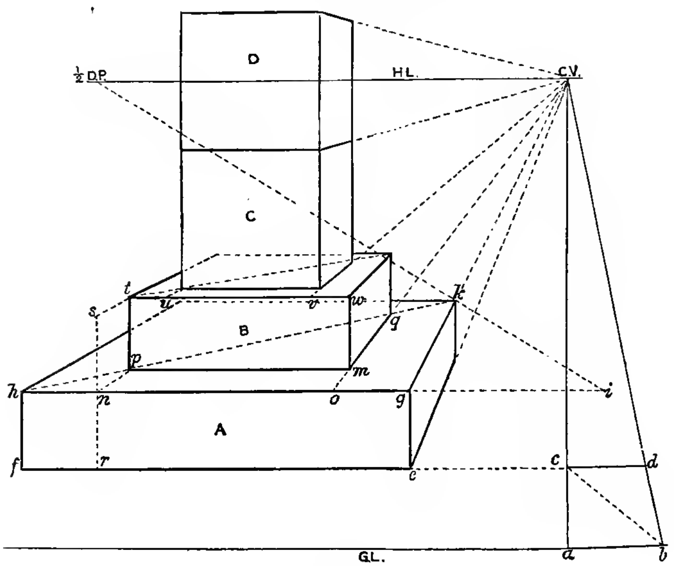


Fig. 52

perspective foot, and complete the front of A. Draw the receding lines to C.V.

To measure 5′ on the receding line from g, set off $gi = 2\frac{1}{2}$ times cd, and draw i to $\frac{1}{2}$ D.P., then $gk = 5'$. Complete A and draw the diagonal hk. As B is 2′ smaller than A, there will be a margin of one foot round B.

Set off hn and go, each equal to cd, and draw lines from n and o to C.V. to meet the diagonal hk in p and q. Draw the horizontal line pm.

To determine the height of B, draw a perpendicular $ns = nr$,

and draw *s* to C.V. to meet a perpendicular on *p* in *t*. Complete B and draw a diagonal as before.

As the width of B is one foot more than C, there will be a margin of half a foot round C. Set off *tu* and *vw*, each equal to half of *pt*, and draw *u* and *v* to C.V. to meet the diagonal. This will determine the width of the cubes, having obtained which we can determine their heights and complete them.

We will now take an illustration in angular perspective [25].

The line E (Fig. 47) is such a line as constantly occurs in angular perspective. We will first draw such a line from given data, as well as a plane containing lines of a similar character; after which we will proceed with rectangular solids at given angles with the P.P.

Problem 11

1. *Draw a line* A *3′ long, lying on the* G.P., *and receding to the left at an angle of* 50°; *its nearer end to be 3′ from* P.P. *and 4′ to spectator's right.*
2. *Also a vertical plane* B, 4′ × 4′, *with one edge resting on the* G.P., *receding to the right at an angle of* 40° *with the* P.P.; *its nearest edge to be 5′ from* P.P. *and 3′ to spectator's left.*

Distance of P.P. = 7′. *Height of eye* = 5′. *Scale* $\frac{1}{4}''$ = 1′. Fig. 53.

1. Determine the G.L., H.L., C.V., P.D., and perspective scale, as in Prob. 5.

Draw a perpendicular from C.V., and make it 7′ long. This will determine the station point S (Fig. 27).

Draw D.L. perpendicular to this line for the directing line [31], and set off an angle of 50° at S with this line, and produce it to meet H.L. in V.P. This is the vanishing point for the line A.

With V.P. as centre, and its distance from S as radius, draw an arc to meet the H.L. in M.P. This is the measuring point for the line A [28].

Set off $ac = 3'$, and draw c to P.D. to meet the line from a to C.V. in d. Draw a horizontal line through d, and set off $df = 4$ perspective feet (*i.e.* four times ed). Draw f to V.P. Set off $fg = 3$ perspective feet, and draw g to M.P., cutting the receding line from f in h. Then fh is the line A.

2. Set off an angle of 40° with D.L. on the right of S, and produce the line to meet H.L. in V.P.′. With this point as

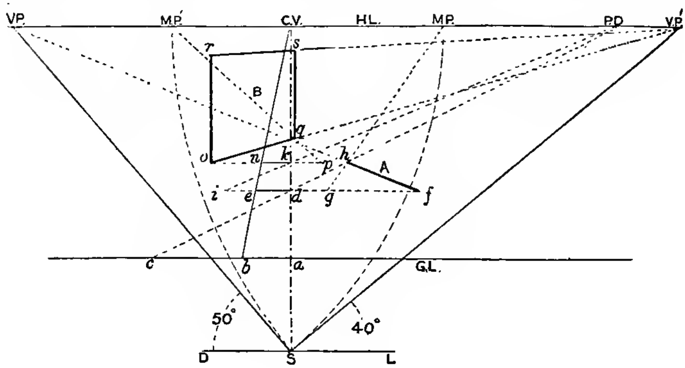


Fig. 53

centre, and its distance from S as radius, draw an arc to meet H.L. in M.P.′

Set off $di = 2$ perspective feet (ed), and draw i to P.D. to meet the line from a to C.V. in k. Draw a horizontal line through k, and set off $ko = 3$ perspective feet (nk), and set up four of the same feet on the perpendicular or. Draw o and r to V.P.′. Set off $op = 4$ perspective feet, and draw p to M.P.′, cutting the receding line from o in q. Draw a perpendicular on this point to meet the receding line from r in s. Then $orsq$ is the plane B.

Problem 12

Draw a rectangular solid of dimensions 3′, 2′, and 1′, *respectively, in angular perspective; its longest edges receding to the right at an angle of* 30°, *and its shortest edges perpendicular to*

the G.P. *Its nearest edge to be* 2′ *from* P.P., *and* 1′ *to the spectator's right.*

Distance of P.P. = 6′. *Height of eye* = 4′. *Scale* ¼″ = 1′. Fig. 54.

Note.—As we shall refer back to this problem in future problems in angular perspective, it will be advisable to describe its construction from the commencement.

Draw G.L. to represent the level of ground plane. Set up 4′ above it for height of spectator's eye, and draw H.L. parallel

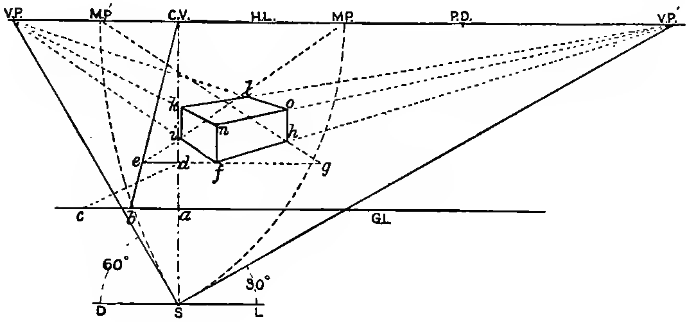


Fig. 54

to G.L. for the horizon line. Assume C.V. to be a point opposite the spectator's eye (the centre of vision), and set off from it, along H.L., 6′, the distance of P.P., for P.D.

Draw a perpendicular from C.V. to meet G.L. in *a*. Set off *ab* = 1 foot, and draw *b* to C.V. This forms the perspective scale.

Produce the perpendicular through *a*, and make its length from C.V. = distance of P.P. (6′); this gives the station point S. Draw D.L. a horizontal line through S, for the directing line [31].

As the edges of the solid receding to the right are at an angle of 30° with P.P., those receding to the left must be at an angle of 60° with it.

Set off these angles at S, with D.L., and produce the lines to meet H.L. in the points V.P. and V.P.′. With these points as

centres, and their distances from S as radii, draw arcs to meet H.L. in the points M.P. and M.P.′.

Set off $ac = 2'$, and draw a line from c to P.D., cutting the line from a to C.V. in d. Draw a horizontal line through d, and set off $df = de$, the perspective foot at this distance. Draw a perpendicular $fn = dc$. Draw lines from f and n to V.P. and V.P.′. Set off $fg = 3$ perspective feet, and draw a line from g to M.P.′, cutting the receding line from f in h. Draw a vertical line at this point, meeting the receding line from n in o. Find the left-hand side of the solid in a similar manner and determine the points i and k. Draw lines from k to V.P.′, and from o to V.P., meeting in l. This completes the solid required.

Half-Station Point

We have already shown the advantage of working with a half-distance point in Prob. 9; we will now carry it a step farther, and determine the V.P.'s and M.P.'s by means of a station point at half of the specified distance of the spectator from the P.P., which is called a *half-station point.*

Problem 13

Draw the model shown in Fig. 55, *built up of cubes of* 1′ *edge and half cubes, using a half-station point.*

The front of the model is to form an angle of 40° *with the* P.P. *on the right; its nearest corner is to be* 2′ *from the* P.P. *and* 1′ *to the spectator's left.*

Distance of P.P. = 6′. *Height of eye* = 3′. *Scale* ½″ = 1′. Fig. 55.

Find the G.L., H.L., C.V., and perspective scale (Prob. 12).

As the distance of P.P. = 6′, set up a perpendicular at C.V. 3′ long, which gives the point s, the *half-station point.* Draw dl (the directing line) perpendicular to this line through s.

As the front of the model forms an angle of 40° with the P.P. on the right, one side of it must form an angle of 50° with

it on the left. Set off these angles with *dl* at *s* by lines meeting the H.L. in the points *v.p.* and *v.p.'*: these are *half-vanishing points.* To determine the true vanishing points, double the distances of these points on H.L. from C.V. The true vanishing points fall beyond the limits of the drawing.

With *v.p.* and *v.p.'* as centres, and their distances from *s* as

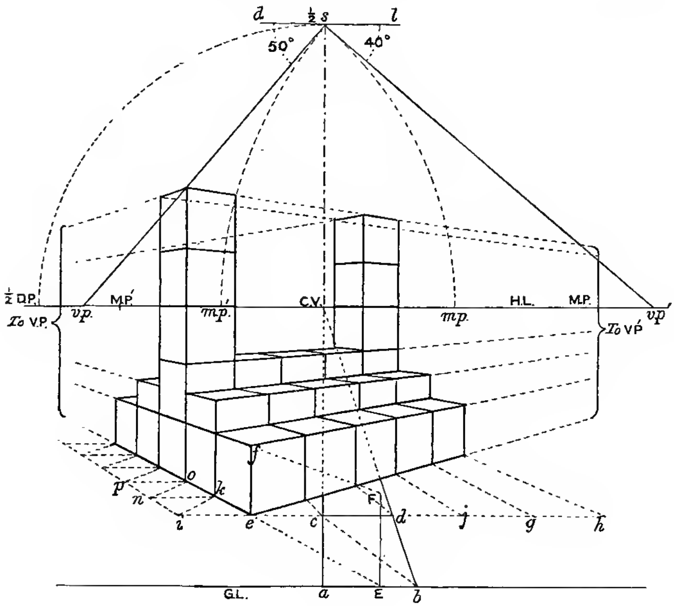


Fig. 55

radii, describe arcs meeting the H.L. in the points *m.p.* and *m.p.'*. Double the distances of these points from C.V. on the H.L., and obtain M.P. and M.P.'. These are the true measuring points.

This method of obtaining the working points is very convenient, especially when the space at our disposal is limited. In very large drawings we could take *s* at one-quarter of the spectator's distance from the P.P., or any other fraction of it to suit our convenience, and determine the true points afterwards as indicated above.

With C.V. as centre, and its distance from *s* as radius, draw

an arc to meet the H.L. in $\frac{1}{2}$ D.P. This is the half-distance point (Prob. 9).

Draw a line from *b* to $\frac{1}{2}$ D.P., meeting the line from *a* to C.V. in *c*. Draw a horizontal line through *c*, and set off $ce = cd$. Also draw a vertical line $ef = cd$, and draw lines from *e* and *f* to the true vanishing points.

Two methods are here explained for measuring the receding edges of the cubes, either of which can be used. We will measure those on the right by one method, and those on the left by another.

(1) To measure those receding to the right, mark off the points *j*, *g*, and *h*, a perspective foot (*cd*) apart, and draw lines from them to M.P.′ to meet the receding line from *e*, and then draw vertical lines to meet the receding line from *f*.

(2) The receding widths of the cubes on the left we will measure as follows :—Set off $ei = cd$, and draw a line from *i* to the true V.P. Join *i* with M.P., meeting the receding line from *e* in *k*. Draw a horizontal line from *k* to meet the line from *i* to V.P. in *n*. Then join *n* to M.P., and proceed in the same manner as before, till five receding feet are measured on the bottom of the model. Draw vertical lines at these points for the sides of the cubes on the left, and complete the model as shown.

Problem 14

1. *Draw a square* $5' \times 5'$ *inclined to the* G.P. *at an angle of* 45° *on the left, with one of its edges resting on the* G.P., *and at right angles to the picture plane; the nearest point of this edge to be* 2′ *from* P.P. *and* 1′ *to spectator's left.*
2. *Draw a line* 5′ *long, perpendicular to this plane, and passing through its centre; its lower end to touch the* G.P.

Distance of P.P. $= 12'$. *Height of eye* $= 4'$. *Scale* $\frac{1}{3}'' = 1'$. Fig. 56.

1. Find the G.L., H.L., C.V., and perspective scale, as in Prob. 5, also the $\frac{1}{2}$ D.P., as in Prob. 9.

Find *d*, the nearest end of the edge *de* (Prob. 10), and join it to C.V. Draw $df = 2\frac{1}{2}$ times *cj*, and join *f* to the $\frac{1}{2}$ D.P., meeting the receding line from *d* in *e*. Then $de = 5'$.

Draw *dg* at an angle of 45° with the G.P. on the left, and five

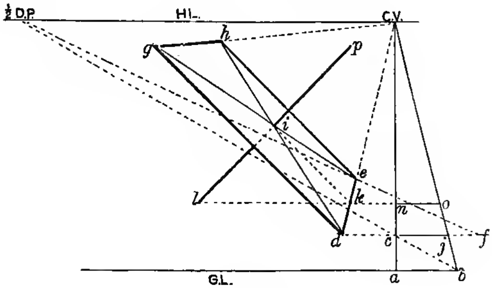


Fig. 56

times *cj* in length. Join *g* to C.V. Draw *eh* parallel to *dg*. Then *dghe* is the square required.

2. Draw diagonals to the square, intersecting in *i*. Draw *ik* parallel to *dg*, and draw a horizontal line through *k*. Draw a line through *i*, perpendicular to *ik*, to meet this line in *l*. Make *lp* five times *no* in length. This is the line required.

EXERCISES

Practice

The scale to be used in working all these problems is half an inch to one foot.

The eye is to be, in each case, 11′ distant from the P.P., and 5′ above the G.P.

1. Represent in perspective a point, A, on the G.P., 10′ to the left of the centre, and 100′ within the picture; and a second point, B, 22′ above the G.P., 35′ to the right of the centre, and 80′ within the picture.

Note.—"To the left or right of the centre" means to the left or right of the centre of vision, and would be measured in the same manner as if described as being to the spectator's left or right.

2. Draw a line 7′ long, lying on the G.P., at right angles to the P.P.; its nearer end being 4′ to spectator's left and 2′ from the P.P.

3. Draw a line 5′ long, parallel to both the P.P. and the G.P.; its right-hand end to be 2′ to spectator's left, 4′ from P.P., and 3′ above the G.P.

4. Draw a line 3′ long, parallel to the P.P., and inclined to the G.P. at an angle of 30°; its left-hand end to rest on the G.P., 2′ from P.P. and 4′ to spectator's left.

5. Draw a line 6′ long. It lies upon the G.P. It inclines to the right at an angle of 50° with the P.P. Its nearer extremity is 3′ from P.P. and 4′ to spectator's left.

6. A rectangle is 4′ × 3′. Draw it in perspective with one of its longer edges resting on the G.P., at right angles to the P.P., and with its shorter edges vertical. Its nearest edge is to be 3′ from P.P., and 5′ to spectator's left.

7. Draw a rectangle 5′ × 3′, lying on the G.P., with its longer edges inclined to the left at an angle of 35° with the P.P., and its nearest angle 3′ to spectator's right and 4′ from the picture.

8. The dimensions of a rectangular solid are 5′, 3′, and 2′. Draw it in perspective, resting on the G.P., with its longest edges vertical; its shortest edges inclined to the right at an angle of 50° with the P.P., and its nearest edge symmetrically opposite to the spectator and touching the P.P.

9. Draw the solid of the preceding question resting on the G.P. on one of its longest edges, which is at right angles to the P.P.; its shortest edges are to be inclined to the G.P. at an angle of 30° on the left, and the nearer extremity of the edge that rests on the G.P. is to be 3′ from the P.P., and 5′ to the spectator's left.

10. A rectangular stone slab, 8′ long, 6′ wide, and 1′ thick, lies with one of its largest faces on the ground. The longest edges are inclined to the P.P. at an angle of 60° towards the right, and the nearest vertical edge is 1′ to the left of the spectator, and 2′ from the P.P. Through the centre of the slab is cut a square hole 3′ × 3′, the sides of which are parallel to the vertical faces of the slab. Represent the slab in perspective.

Theory

1. What is a perspective scale? Explain the advantages to be derived from its use (Prob. 5).

2. What are half-distance points? Explain their use (Prob. 9).

3. What is a perspective foot? If a line a foot long is placed in perspective parallel to the picture plane, and at the same distance from it as the spectator, what proportion does it bear to a scale-foot (Prob. 5)?

4. Explain the use of a half-station point (Prob. 13).

CHAPTER IV

TRIANGLES, POLYGONS, PRISMS, AND PYRAMIDS

PROBLEM 15

Draw an equilateral triangle of $5\frac{1}{2}'$ *side, lying on the* G.P., *with one of its sides inclined to the right at an angle of* 50° *with the picture plane, and its nearest angle* 2′ *from the* P.P. *and* 3′ *to spectator's right.*

Distance of P.P. = 8′. *Height of eye* = 4′. *Scale* $\frac{1}{4}''$ = 1′. Fig. 57.

Find the G.L., H.L., C.V., P.D., D.L., station point, and perspective scale, as in Prob. 12.

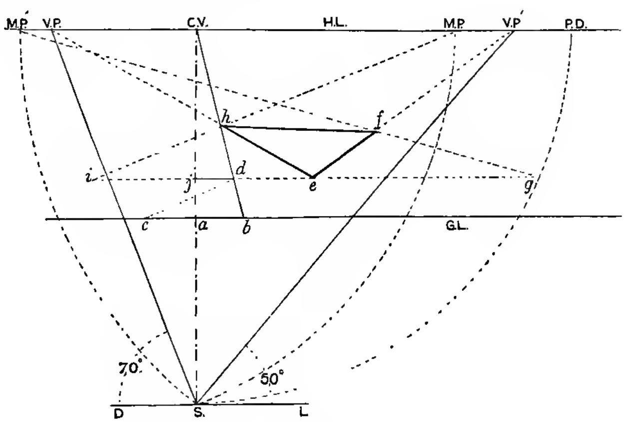


Fig. 57

If one side of the triangle inclines to the right at an angle

of 50°, the angles of an equilateral triangle being 60°, another side must incline to the left at an angle of 70° with the P.P., for the sum of the three angles must be 180°.

Set off these angles at S, with D.L., and find V.P. and V.P.′; also M.P. and M.P.′ (Prob. 12).

Find *e*, the nearest angle of the triangle (Prob. 12).

Join *e* to V.P. and V.P.′. Draw a horizontal line through *e*, and set off *ei* and *eg*, each equal to five and a half times *jd*—the size of a perspective foot at this distance—and draw a line from *i* to M.P. to meet the receding line from *e* to V.P. in *h*. Find the point *f* in a similar manner, and join *hf*.

Then *ehf* is the triangle required.

Problem 16

1. *Draw an equilateral triangle of* 6′ *edge, perpendicular both to the* P.P. *and the* G.P., *with one of its edges resting on the* G.P., *and the nearer end of this edge* 4′ *from the* P.P. *and* 4′ *to spectator's right.*
2. *Draw the same triangle inclined to the* G.P. *at an angle of* 45° *on the left, with one of its edges resting on the* G.P. *perpendicular to the* P.P., *and the nearer end of this edge* 2′ *from the* P.P. *and* 3′ *to the spectator's left.*

Distance of P.P. = 14′. *Height of eye* = 4′. *Scale* $\frac{1}{4}'' = 1'$. Fig. 58.

1. Find the G.L., H.L., C.V., and perspective scale, as in Prob. 5, and the ½ D.P., as in Prob. 9.

Find the point *f* (Prob. 10), and draw a line through it from C.V. to meet G.L. in *m*; also draw a horizontal line from it through the perspective scale. Draw a line from *f* to ½ D.P. to meet the receding line from *e* in *g*. As *ef* = 3′, *eg* must equal 6′. Draw a horizontal line from *g* to meet the receding line from *f* in *h*.

To find the centre of the line *fh*, bisect *ef* in *j* and draw *j* to

$\frac{1}{2}$ D.P. to meet *eg* in *i*. Draw a horizontal line from *i* to meet *fh* in *k*.

In any convenient position draw the equilateral triangle (ABC) geometrically. It need not be drawn full size. In this instance it is drawn half size. Draw the perpendicular CD.

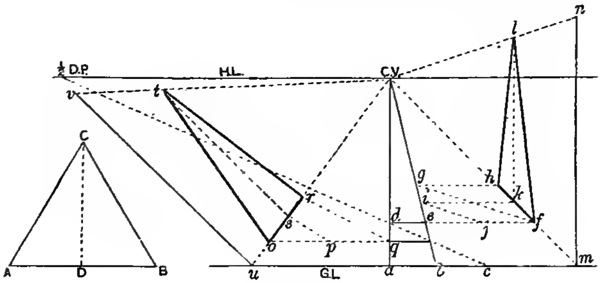


Fig. 58

Draw a vertical line *mn* at *m* twice the height of DC. Draw *n* to C.V. cutting a perpendicular on *k* in *l*. Join *lh* and *lf*. Then *fhl* is the triangle required.

2. Find the edge *or*, and bisect it perspectively in *s* by the method just described.

Draw *uv* at an angle of 45° with G.L. and equal in length to twice CD ; also draw a line from *s* parallel to it. Join *v* to C.V., meeting this line in *t*. Join *to* and *tr*. Then *ort* is the triangle required.

Problem 17

Draw an equilateral triangular prism 9′ long, and with each face 4′ wide, lying with one of its faces on the G.P., *and its longer edges inclining to the left at an angle of* 40° *with the picture plane ; its nearest corner is to be 3′ from the* P.P. *and 3′ to the spectator's right.*

Distance of P.P. = 11′. *Height of eye* = 5′. *Scale* $\frac{1}{6}'' = 1'$. Fig. 59.

Find all the necessary working points and lines, as well as the nearest corner (*e*) of the object, as in Prob. 12.

Draw *e* to each V.P. Draw a horizontal line through *e* and set off *ef* = 9 times *co*. Draw *f* to M.P. to meet the receding line from *e* in *g*.

From M.P.′ draw a line through *e* to meet the G.L. in A. On the right of this point construct an equilateral triangle

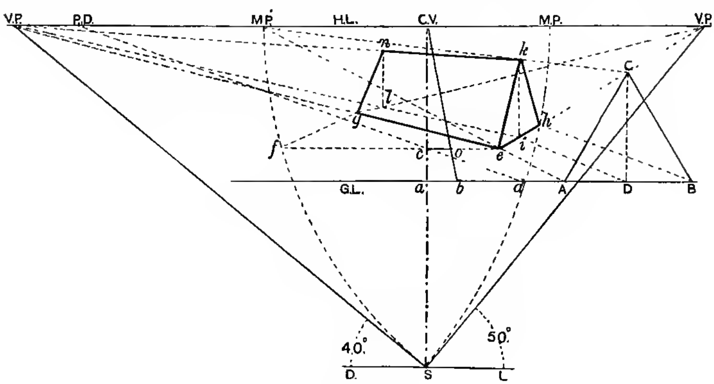


Fig. 59

ABC with sides 4′ long. Draw the perpendicular CD, and join A, D, and B, with M.P.′, by lines meeting the right-hand receding line from *e* in *i* and *h*.

Draw a line from C also to M.P.′, meeting a vertical line on *i* in *k*. Join *ke* and *kh*.

Draw a line from *g* to V.P.′ to meet a line drawn from *i* to V.P. in *l*. Draw *k* to V.P. to meet a perpendicular on *l* in *n*. Join *ng*. This completes the prism.

Problem 18

Draw the regular pentagon ABCDE *of* 5′ *side in perspective without using* V.P.'s. *The pentagon is to lie on the* G.P. *with the side* AB *inclined to the right at an angle of* 25° *with the picture plane. The angle at* A *is to be* 2′ *from the* P.P. *and* 2′ *to the spectator's left.*

Distance of P.P. = 16′. *Height of eye* = 8′. *Scale* $\frac{1}{8}'' = 1'$. Fig. 60.

Draw AF at an angle of 25° with AB, 2′ long. Draw FS perpendicular to AF, and set off 2′ from F, and draw H.L. through the point, perpendicular to FS. Set off from H.L. on FS, 16′, to determine the point S. Draw G.L. parallel to H.L. and 8′ below it.

Assume H.L. to be the position, in plan, of the P.P., and draw visual rays from each corner of the pentagon to S, meeting the H.L. in the points *e*, *d*, *a*, *c*, *b*. Draw perpendiculars from the points of the pentagon to meet the G.L. in the points E′, D′, A′, C′, B′. To avoid confusion only one line is drawn (EE″E′). Draw lines from these points to C.V., *e.g.* E′*e*′ is the perspective of the line E″E. These lines meet the corresponding perpendiculars from the points *e*, *d*, *a*, *c*, *b*, in the points *e*′, *a*′, *b*′, *c*′, *d*′. Join these points, as shown.

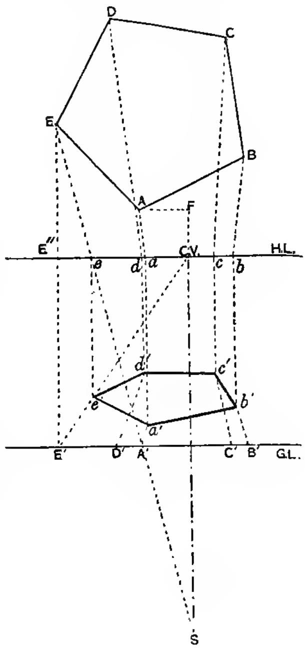


Fig. 60

Problem 19

Draw a regular hexagon of 3′ side, lying on the G.P., *with one of its sides parallel to the* P.P., *and its nearest right-hand corner 3′ from the* P.P. *and* 1′ *to the spectator's right.*

Distance of P.P. = 8′. *Height of eye* = 4′. *Scale* $\frac{1}{4}'' = 1'$. Fig. 61.

Find all the necessary working lines and points (Prob. 12), and the nearest right-hand corner (*e*) (Prob. 10).

In this problem it will be seen that each V.P. is the M.P. for the other.

Draw a horizontal line through *e*, and make *ej* = 3 times *ce*.

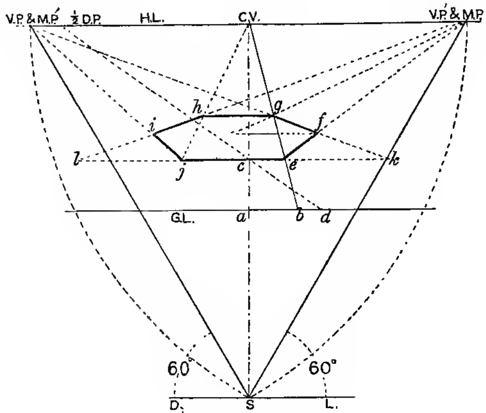


Fig. 61

Draw *e* to V.P.′, and *j* to V.P. Make *ek* and *jl* each equal to *je* and join them to the opposite M.P.'s to meet the receding lines drawn from *e* and *j* in *f* and *i*. Draw lines from *f* and *i* to the opposite V.P.'s to meet lines drawn from *e* and *j*, to C.V., in the points *g* and *h*. Join *hg* to complete the polygon.

Problem 20

Draw a hexagonal prism $3\frac{1}{2}'$ *long, with sides* $2'$ *wide, resting on one of its shorter edges, and inclined to the* G.P. *at an angle of* 60° *on the right; the edge upon which it rests is to be perpendicular to the* P.P., *and its nearer end* $4'$ *from the* P.P. *and* $6'$ *to the spectator's left.*

Distance of P.P. $= 19'$. *Height of eye* $= 6'$. *Scale* $\frac{1}{4}'' = 1'$. Fig. 62.

Find the necessary working points and lines, as in Prob. 5; the ½ D.P., and the point *c* (Prob. 10).

In any convenient position on the G.L. draw the hexagon

ABCDEF with 2′ sides; and draw perpendiculars from the several points to meet the G.L. in the points F′, A, B, and C′. Assume any point *v* on the H.L. and join these points to it, meeting a horizontal line through *c* in the points *k*, *l*, *n*, *o*.

Draw a horizontal line from the angle C of the hexagon to meet the perpendicular BD in G.

Draw a line from C.V. through *c*, meeting the G.L. in *h*. Draw *hj* at an angle of 30° with the G.L., and set off upon it the distances *hi* and *ij*, equal to BG and GD.

Draw a line at *c* at an angle of 60° with *cb*, and equal in

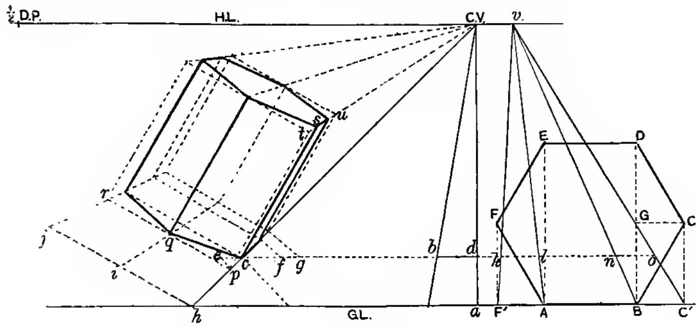


Fig. 62

length to three and a half times *bd*. This gives point *s*. Set off *ce* equal to half of *kl*, and draw a line from ½ D.P. through *e* to meet the receding line from *h* in *p*. Also set off *cf* and *fg*, equal to half of *ln*, and *no*, and draw lines from *f* and *g* to ½ D.P. to meet the same line. Draw lines from these points parallel to *cs* to meet a line drawn from C.V. through *s*. Also draw lines from the same points parallel to *hj* to meet lines drawn from *i* and *j* to C.V. Complete the prism, as shown.

Problem 21

Draw a hexagonal prism 6′ long, with sides 2′ wide, lying on the G.P., with its longer edges receding to the right at an angle

of 55° with the picture plane, and its nearest angle on the G.P., 2′ from P.P. and 2′ to spectator's left.

Distance of P.P. = 10′. *Height of eye* = 6′. *Scale* $\frac{1}{6}'' = 1'$. Fig. 63.

Find all the necessary working lines and points, as well as the point *c* (Prob. 12).

Draw lines from *c* to each V.P. In any convenient position on the G.L. construct the hexagon ABCDEF with 2′ sides, and draw perpendiculars from the corners to meet the G.L. in the points H, A, B, and K. Take any point *v′* on H.L., and to it

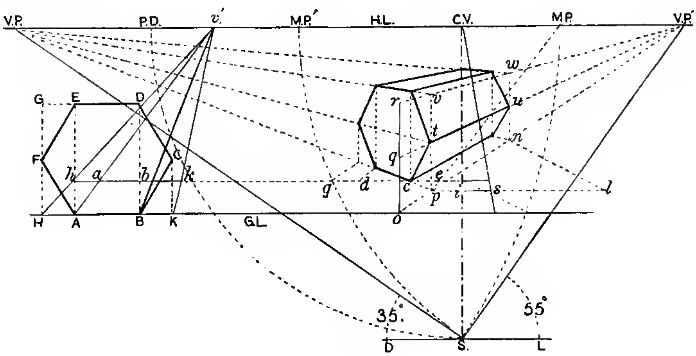


Fig. 63

draw lines from these points, meeting a horizontal line through *c* in the points *h*, *a*, *b*, *k*.

Set off *ce* equal to *bk*, and draw a line from M.P. through *e* to meet the line drawn from V.P. through *c* in *p*. Set off *pl* equal to six times *is*, and draw a line from *l* to M.P.′, meeting a line drawn from *p* to V.P.′ in *n*. Produce *np* to meet the G.L. in *o*, the point of contact [49]. Set up at *o* a line of heights [46], and make *oqr* equal to HFG. Draw lines from *q* and *r* to V.P.′ to meet vertical lines at *p* and *n* in the points *tv* and *uw*. Set off from *c*, *cd*, equal to *ba*, and *dg* = *ah*, and draw lines from *d* and *g* to M.P., meeting the receding line from *p* to V.P. Draw vertical lines at these meeting points to meet lines drawn from *t* and *v*

to V.P. Join the points as shown, and proceed in a similar manner with the opposite end, to complete the prism.

Problem 22

1. *Draw an octagonal prism 6′ long, with sides 15″ wide, lying with one of its faces on the* G.P., *and its longer edges receding to the left at an angle of* 35° *with the picture plane; its nearest angle on the* G.P. *being 2′ from the* P.P. *and 3′ to the spectator's right.*
2. *Draw a line, indefinite in length, at right angles to the nearer upper edge of the prism, and passing through its middle point. The lower end of this line is to meet the* G.P. *4′ from the nearer lowest edge of the prism.*

Distance of P.P. = 12′. *Height of eye* = 5′. *Scale* $\frac{1}{3}'' = 1'$. Fig. 64.

1. In this problem we will take the station point at one-third of its specified distance (Prob. 13).

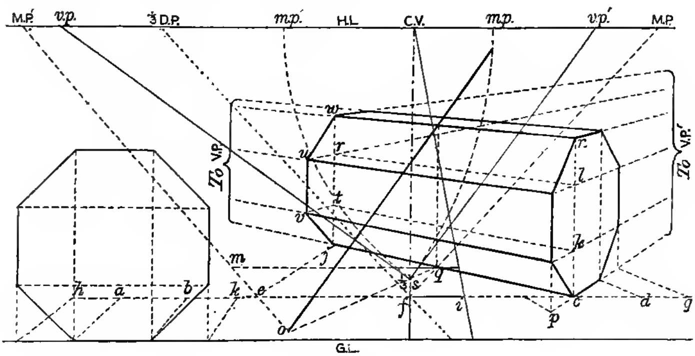


Fig. 64

Find all the necessary working lines and points (Prob. 12), and the point *c* of the figure (Prob. 10).

Draw an octagon of the required size and obtain the points

h, *a*, *b*, *k*; also the points *p*, *c*, *d*, *g*, as by the methods described in the preceding problem.

Join *c* to each V.P. Set off *ce* equal to six times *fi*, and draw *e* towards M.P., meeting the receding line from *c* in *j*. Draw lines from *d* and *g* to M.P.′, and where they meet the receding line from *c*, draw vertical lines, as well as on the points *c* and *p*.

Mark off the distances *ck*, *kl*, *ln*, equal to the distances *ha*, *ab*, *bk*, and draw lines through *k*, *l*, *n*, to V.P.′, meeting the other vertical lines on the right of the prism. Join the points, as shown, to complete the octagon in perspective.

Draw lines from *k*, *l*, *n*, to V.P., meeting the vertical line at *j* in the points *t*, *r*, *w*. Draw lines from V.P.′ through these points to meet lines drawn from the right-hand end of the prism to V.P. in the points *u* and *v*. Join the points, as shown, to complete the prism.

2. Since $fc = 3'$, *f* joined to M.P. will give the middle of the line *cj* in *q*; and a vertical line at *q* will give the centre of the corresponding upper edge of the prism.

Draw the horizontal line *qm*, four perspective feet in length, and draw a line from M.P.′, through *m*, to meet in *o* a line drawn from V.P.′ through *q*. This is the lower end of the line required. Draw the line from *o* through the centre of the upper edge of the prism already found.

Problem 23

1. *Draw a right pyramid* 6′ *high, with a base* 3′ *square, standing on the* G.P., *with its sides receding at an angle of* 45° *with the* P.P.; *its nearest angle being* 2′ *from the* P.P. *and* 4′ *to the spectator's right.*
2. *Draw an equal pyramid lying with one of its sides on the* G.P., *and its base perpendicular to the* P.P.; *its nearest angle on the* G.P. *being* 3′ *from the* P.P. *and* 5′ *to the spectator's left.*

Distance from P.P. $= 10'$. *Height of eye* $= 5'$. *Scale* $\frac{1}{4}'' = 1'$. Fig. 65.

1. Find the necessary working lines and points, as well as the point *e* (Prob. 5).

As each edge of the base is inclined at an angle of 45° with the P.P., the P.D. will be one of its vanishing points.

Find its M.P. (Prob. 12). Draw *e* to V.P. Set off $ei = 3$

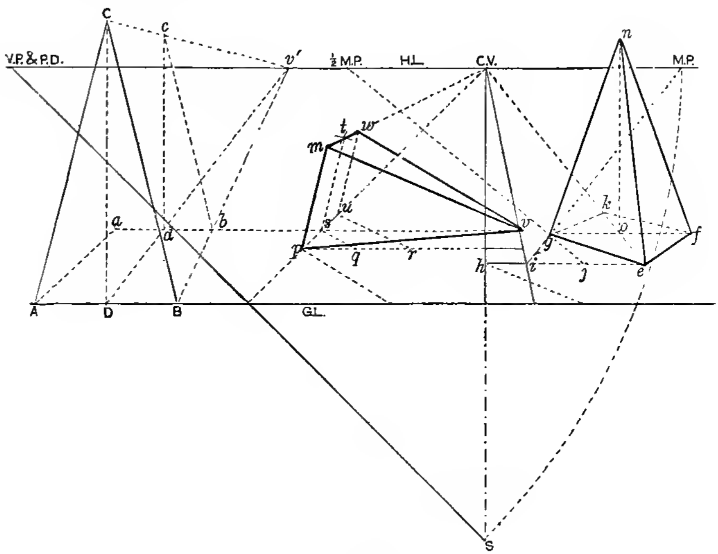


Fig. 65

times *hi*, and join *i* to M.P., meeting the receding line from *e* in *g*. Draw the horizontal line *gf*. Join *e* to C.V., cutting this line in *o*. Make *of* equal to *go*, and join *f* to V.P., meeting *eo* produced in *k*. Join *gk*. Then *gefk* is the base of the pyramid. Observe that we have been able to draw this without using the second V.P.; because *gf*, being parallel to the P.P., we could determine the point *f* by making $of = go$.

Draw the vertical line *on*, six perspective feet in height, and join *n* to the corners of the base to complete the pyramid.

2. In any convenient position on the G.L. construct ABC, a

geometrical elevation of the pyramid. Assume any point v' on the H.L., and draw lines to it from the points ADBC.

Find the point p (Prob. 5), and join it to C.V. Draw a horizontal line through p. Set off $pr = 3$ perspective feet, and bisect pr in q. Draw lines from q and r to P.D., meeting the receding line from p in s and u. Draw a horizontal line through s and determine the points a, d, b, and c, as shown in the figure. Make $sv = bc$. With v and s as centres, and radii equal, respectively, to bc and ba, draw arcs cutting each other in t. Draw a line from C.V. through t to meet lines drawn from p and u, parallel to st, in m and w. Join p, m, and w to v to complete the pyramid.

Proportional Measuring Points

A *proportional measuring point* is a similar device in angular perspective to the *half-distance point* in parallel perspective (Prob. 10), *e.g.* to find a *half-measuring point*, bisect the distance between V.P. and M.P. This gives us the $\frac{1}{2}$ M.P. To use this point for measuring the line ge (Fig. 65), bisect ei in j. Join j to the $\frac{1}{2}$ M.P. This also gives the point g.

A measuring point for any other proportion could be found in a similar manner.

Problem 24

Let Fig. 66 *represent, to the scale of* $\frac{1}{4}'' = 1'$, *the front and side elevation of a rectangular solid pierced by a hexagonal pyramid. Draw this model to twice this scale in perspective, with the axis of the pyramid parallel to the* P.P., *and the point* A 1′ 8″ *from the* P.P. *and* 7′ *to the spectator's right.*
Distance of P.P. = 13′. *Height of eye* = 4′. Fig. 67.

Find the necessary working lines and points (Prob. 5), the $\frac{1}{2}$ D.P. and the point A (Prob. 10).

Draw a line from C.V. through A till it meets the G.L. in a. Set off aC′ = twice A′C′ (Fig. 66), and join C′ to C.V. Set

off from C′ the distance C′A′ equal to half the distance of A from the P.P., and set off from A′ the points B, C, D, E, F, G, at distances equal to the distance of the corresponding points in

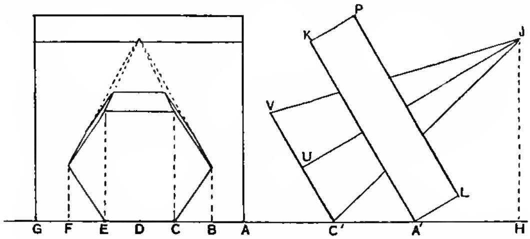


Fig. 66

Fig. 66, and join them to the ½ D.P. by lines meeting the receding line from C′ in the points *b*, *c*, *d*, *e*, *f*, *g*. Draw the line C′V parallel to C′V in Fig. 66, but of double its length, and bisect it in U. Draw lines from U and V to C.V., meeting lines drawn from the points *b*, *c*, *e*, *f*, parallel to C′V, in the

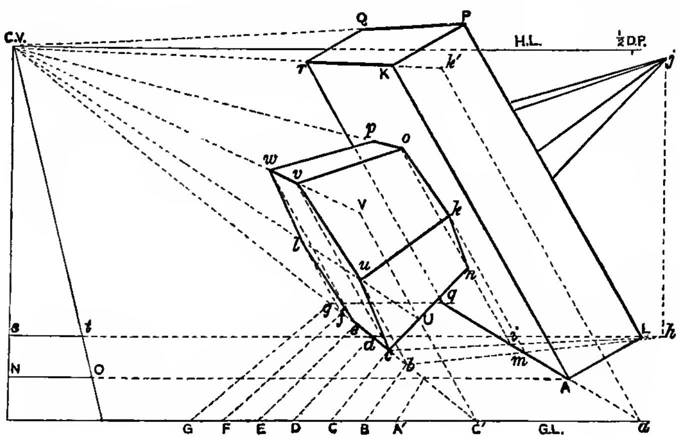


Fig. 67

points *u*, *v*, *w*, *l*. Join these points, as shown, to complete the base of the pyramid.

Draw *ak′* parallel to C′V, and equal to twice A′K (Fig. 66). Join *k′* to C.V., meeting a line from A, drawn parallel to *ak′* in

K. Draw AL perpendicular to AK and equal in length to the perspective foot NO.

Draw KP equal and parallel to AL, and join PL. Join K and P to C.V. Draw a horizontal line from *g* to meet the receding line from A in *q*. Draw *qr* parallel to AK, and *r*Q parallel to KP. Draw a horizontal line through *d*. As the point H (Fig. 66) is 4′ from C′, make $dh = 4$ times *st*; and a perpendicular at *h* 3′ 10″ high by the same perspective foot, to correspond with the height of J (Fig. 66).

Join *b* and *c* to *h*, cutting the receding line from A in *m* and *i*. Draw lines from *c*, *u*, and *v*, to *j*, to meet lines drawn from *m* and *i*, parallel to AK, in *k*, *n*, and *o*. Draw a line from *o* to C.V. to meet a line drawn from *w* to *j* in *p*. Join the points, as shown, to complete the figure.

EXERCISES

Practice

The scale to be used in working all these problems is $\frac{1}{3}$ inch to 1 *foot. The eye is supposed to be, in each case,* 12′ *distant from the* P.P., *and* $5\frac{1}{2}$′ *above the* G.P.

1. A square right pyramid of 3′ base and 6′ axis stands with its base on the G.P. One diagonal of its base is parallel to the picture, and its nearest corner is 5′ to the spectator's right and 4′ from the G.L.

2. A right line, 7′ in length, lies upon the G.P., and is at right angles to the G.L. The nearer end of the line is 3′ on the left of the spectator, and 2′ from the G.L. This line is one side of an equilateral triangle, whose plane is inclined to the G.P. at an angle of 60° towards the left. Represent the triangle in perspective.

3. A right line 3′ long is lying on the G.P., inclined to the right at an angle of 40° with the P.P.; its nearer end is 2′ from the P.P. and 4′ to the spectator's right. This line is the nearest right-hand side of a regular hexagon lying on the G.P. Draw the hexagon in perspective.

4. Draw a square right pyramid of 3′ base and 6′ axis, lying with one of its sides on the G.P., and having its axis parallel to the P.P. Its base is towards the centre of the picture, and its vertex 8′ to spectator's left and 5′ from the G.L.

5. An equilateral triangle of 5′ side is in a vertical plane inclined to the left at an angle of 35° with the P.P., and has one side resting on the G.P. The triangle is one end of a right prism which has an axis of 7′. Draw the prism with its nearest corner 3′ from the P.P. and 2′ to the spectator's left.

6. Let the line AB (Fig. 60) represent the nearest side of a hexagon parallel to the P.P., the point A being 2′ from the P.P. and 1′ to the spectator's right. Let the hexagon be the base of a pyramid, having an axis of 10′. Draw it in perspective.

7. The hexagonal prism (Fig. 62) is $3\frac{1}{2}$′ long, with sides 2′ wide. Draw this prism with one of its faces on the G.P., and its longer edges receding to the right at an angle of 30° with the P.P. The nearest corner of its lowest face is to be 3′ from the G.L. and 1′ to the spectator's left. Draw a line, of indefinite length, passing through the centre of the nearer upper edge of those receding to the right, and at right angles to it, and meeting the G.P. in a point 4′ from the nearer lower edge of the prism.

8. Draw in perspective the object represented in Fig. 66 to twice the scale there shown. The base of the pyramid is to rest on the G.P. The point U is to be 3′ from the P.P. and 2′ to the spectator's left. The edge KA′ of the rectangular solid is to recede to the right at an angle of 50° with the P.P.

9. Represent in perspective a line 2′ long on the G.P. inclined at 40° to the G.L. towards your left, and having its nearest point opposite the spectator and 2′ from the G.L. Make this line one of the near edges of an octagonal slab, 1′ thick, lying flat upon the G.P., and draw any line in a plane at 45° to the P.P. to meet the G.P., and pass through one of the upper edges of the slab.

10. Represent in perspective a line AB, having one end, A, upon the ground, 3′ to the right of the centre, and 3′ from the G.L.; and the other end, B, 3′ above the ground, 3′ to the left, and 9′ within the picture. Upon this line AB construct in perspective a regular hexagon, having one side resting on the ground, bisected at point A, and inclined at 45° towards the right.

Theory

1. What is the difference between a vanishing point and a measuring point [13 and 28]?

2. What is the difference between a vanishing line and a picture line [33 and 40]?

3. What are proportional measuring points (p. 80)?

4. What is a vanishing point? Why is it so called [13]? Why are some vanishing points called *accidental* [39]?

CHAPTER V

CIRCLES AND OTHER CURVES, CYLINDERS, AND CONES

Problem 25

1. *Draw a cube of 3′ edge in parallel perspective, so that its nearer right-hand edge may be 2′ from the* P.P. *and 7′ to the spectator's left. Inscribe circles in the squares forming its visible faces.*
2. *Draw upon the* G.P. *four circles each of 3′ diameter, and touching the next, so that their centres may be on a line perpendicular to the* P.P. *and* $4\frac{1}{2}'$ *to the spectator's left. The nearest circle is to be 2′ from the* P.P.

Distance from PP = 16′. *Height of eye* = 6′. *Scale* $\frac{1}{3}'' = 1'$. Fig. 68.

1. Find the usual working lines and points (Prob. 5), the $\frac{1}{2}$ D.P. and the point *a* (Prob. 10).

First draw the cube in perspective by the method shown in Prob. 10.

Draw diagonals to the front face of the cube, intersecting each other in *c*. Draw a horizontal and vertical line through this point, meeting the sides of the square in the points *tr* and *sq*. Draw similar lines on the side and top of the cube. With *c* as centre, and radius *ct*, describe a circle, cutting the diagonals in the points *e*, *f*, *g*, *h*. Draw horizontal lines through *ef* and *hg* to meet the nearer side of the square in *d* and *b*.

Draw lines from *d* and *b* to C.V. to cut the diagonals on the side of the cube in the points *l*, *n*, *p*, *o*.

We have now eight points in the right-hand circle, viz. *r*, *l*, *i*, *n*, *j*, *o*, *k*, *p*. Draw a fair curve through these points, taking care to remember that the sides of the square are to be tangents to the curve.

Proceed in the same manner with the top of the cube, and draw a fair curve through the eight points, as shown.

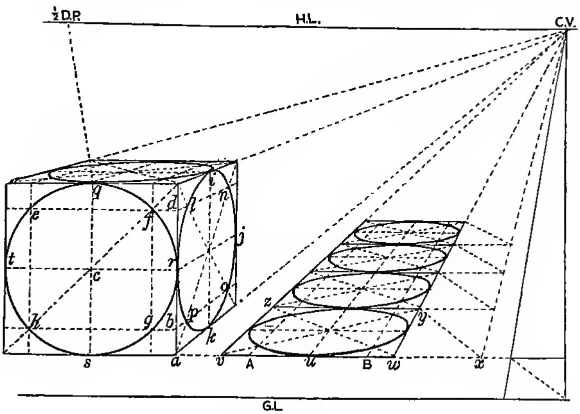


Fig. 68

2. Find the point *u* (Prob. 10). Set off one and a half perspective feet on each side of *u* to determine the points *v* and *w*. Join these points to C.V. Set off *wx* equal to one and a half perspective feet, and join *x* to C.V.

Draw a line from *x* to ½ D.P. to meet the receding line from *w* in *y*. Draw the horizontal line *yz*. Then *vwyz* is a square. In a similar manner draw the three other squares, as shown.

Draw diagonals to each of these squares, and horizontal lines through their centres. Set off a distance equal to *rb* on each side of *u* to find the points A and B, and join them to C.V. We now have eight points in each of the circles. Draw a fair curve through them, as shown, touching the sides of the square.

PROBLEM 26

1. *Draw a hollow cylinder 9′ long, 4′ in external diameter, and 6″ thick; let it lie on the* G.P., *with its axis at right angles to the* P.P., *and the nearer end of the axis 4′ from* P.P., *5′ to the spectator's left, and 2′ above the* G.P.
2. *Place, leaning against the above cylinder, equidistant from its ends and touching it along a horizontal line, a rectangular solid of dimensions 7′, 3′, and 1′, respectively, inclined at an angle of 45° to the* G.P., *upon which one of its 3′ edges rests.*

Distance of P.P. = 20′. *Height of eye* = 6′. *Scale* $\frac{1}{4}''=1'$. Fig. 69.

1. Find the necessary working lines and points (Prob. 5), the $\frac{1}{2}$ D.P. and the point *e* (Prob. 10).

Draw a vertical line at *e* and set off upon it *ec* equal to twice

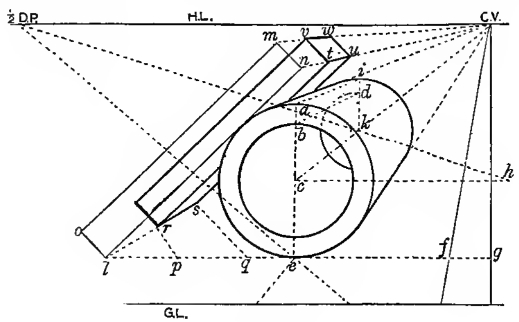


Fig. 69

fg. Join *c* to C.V., and draw the horizontal line *ch* equal to four and a half times *fg*. Draw a line from *h* to $\frac{1}{2}$ D.P. to meet the receding line from *c* in *k*. Then *ck* is the axis of the cylinder.

With *c* as centre, and radius *ce*, describe a circle. Set off *ab* equal to half of *fg*, and describe a concentric circle through *b*. Draw lines from *a* and *b* to C.V., meeting a perpendicular at *k* in *i* and *d*. With *k* as centre describe circles through *i* and *d*, and complete the cylinder, as shown.

N.B.—The outlines parallel to the axis are obtained by drawing tangents from C.V. to either of the outer circles at the ends of the cylinder.

2. Draw the line *ln* a tangent to the larger circle, at an angle of 45° with *lg*, and make it seven times *fg* in length. Draw *lo* and *nm* perpendicular to *ln*, and each equal in length to *fg*. Join *om*.

Join *l*, *n*, *m*, to C.V.

As the cylinder is 9′ long, and the edges of the solid parallel to it are to be 3′ long, there must be a length of 3′ of cylinder on each side of the solid. Set off *lp* and *pq*, each equal to one and a half times *fg*. Draw lines from *p* and *q* to the ½ D.P. to meet the receding line from *l* in *r* and *s*. Then each of these distances, *lr*, *rs*, is 3′.

Draw the lines *rt* and *su* parallel to *ln*, and the lines *tv* and *uw* parallel to *nm*. Draw a line from *v* parallel to *mo*, meeting a line from *r* parallel to *lo*. This completes the solid.

Problem 27

Let ABCD *represent the plan of a folding screen drawn to a scale of* ⅓″ = 1′. *The total height of the screen is to be* 6′, *and each leaf to have a semicircular top. The leaf* B *is to be the farthest from the* P.P., *and parallel to it; and the point* a 2′ *from the* P.P. *and* 4′ *to the spectator's left.*

Distance of P.P. = 7′. *Height of eye* = 4′. *Scale* ⅓″ = 1′. Fig. 70.

Draw the G.L. in any convenient position above B, and parallel to it. Draw the line *ab* perpendicular to G.L. Set off *bc* = 4′, and draw a perpendicular through *c*. Draw H.L. 4′ above GL, and mark off S 7′ from C.V. It will be found by applying the given scale that A, B, C, D, are each 3′.

Draw lines from S parallel to the leaves A, C, and D, to meet H.L. in three points, which will be the V.P.'s for the respective leaves. The distance of S from each V.P., set off in the usual way on the H.L., will give the M.P.'s corresponding to each V.P. (Prob. 12).

Join b to C.V. Set off $bd = 1'$, and draw a line from d to $\frac{1}{2}$ D.P. (Prob. 10), meeting the receding line from b in a'. Draw a' to V.P.A. Draw a horizontal line through a', and set off $a'e = 3$ times gh. Draw a line from e to M.P.A. to meet the receding line from a' in f. Draw a horizontal line through f, and make fj and jk each $= 3$ perspective feet. Draw a line

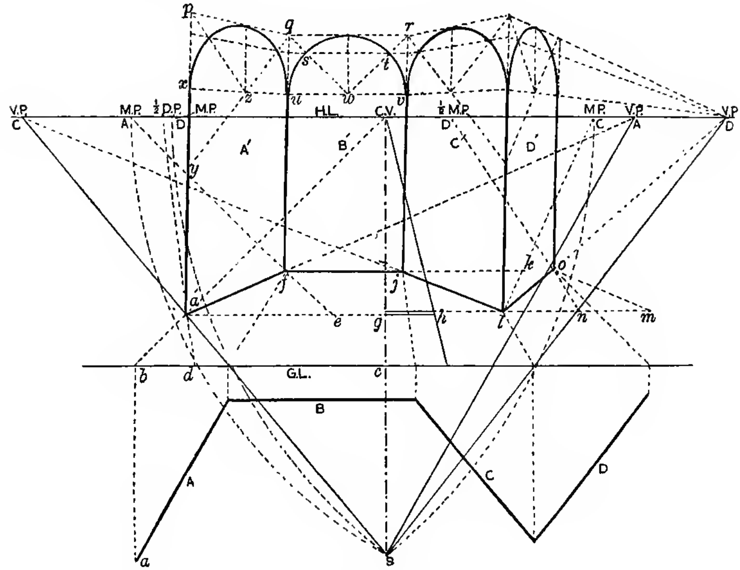


Fig. 70

from M.P.C. through k to meet a line drawn from V.P.C. through j in l.

Draw l to V.P.D., and set off $lm = 3$ perspective feet, and draw a line from m to M.P.D. to meet the receding line from l in o; or $ln = 1\frac{1}{2}$ perspective feet might be set off, and n drawn to $\frac{1}{2}$ M.P.D., which would equally well give us the point o.

When the plan of an object is given, as in this instance, the widths of the several leaves can be determined without using the M.P.'s; *e.g.* project upon the G.L. each of the points on the plan, then join their projections to C.V., as shown.

Set up $a'p = 6$ perspective feet, and draw a line from p to

V.P.A. to meet a perpendicular on *f* in *q*. Draw a horizontal line from *q* to meet a perpendicular on *j* in *r*. Proceed in a similar manner with the leaves C′ and D′.

Set off $qu = 1\frac{1}{2}$ perspective feet. Draw the horizontal line *uv*, and bisect it in *w*. With *w* as centre, and radius *wu*, describe a semicircle. Join *w* to *q* and *r*, cutting the semicircle in *s* and *t*. Draw a horizontal line through *s* and *t*, and produce it towards the several vanishing points through each leaf. Bisect this line on each leaf in the following manner:—Take the leaf A′, for example. Set off *xy* equal to *px*. Join *y* to *q*, cutting the horizontal line in *z*. Draw a vertical line from *z*, and join *z* to *p* and *q*; in this way we can obtain five points in the curve. Proceed in a similar manner with the leaves C′ and D′, and draw the curves, as shown.

Circles in Parallel Planes

Problem 28

1. *Draw a cylinder 9′ long and 4′ in diameter, with its axis perpendicular to the* G.P.; *the point* A *is to be 3′ from* P.P. *and 7′ to the spectator's left. Divide the cylinder into six equal parts by the intersection of five horizontal planes.*
2. *Draw a similar cylinder of the same dimensions, lying on the* G.P. *with its axis inclined to the right at an angle of* 48° *with the* P.P.; *the nearest point* (B) *of the cylinder is to be 3′ from the* P.P., 1′ *to the spectator's right, and* 2′ *above the* G.P. *Divide this cylinder into three equal parts by vertical planes parallel to the ends.*

Distance from P.P. = 14′. *Height of eye* = 6′. *Scale* $\frac{1}{4}'' = 1'$.
Fig. 71.

1. Find the usual working lines and points (Prob. 5), the $\frac{1}{2}$ D.P. and the point A (Prob. 10).

Draw a horizontal line through A, and on it set off two perspective feet on each side of A, obtaining the points *a* and

b. Join these points to C.V. Draw a line from A to ½ D.P. to meet the receding line from *a* in *q*. Draw a horizontal line through *q* to meet the receding line from *b* in *r*. Then *abrq* is a square in perspective. Draw diagonals to the square, and also the horizontal line *eh* through its centre.

With A as centre, and radius A*b*, describe a semicircle, and

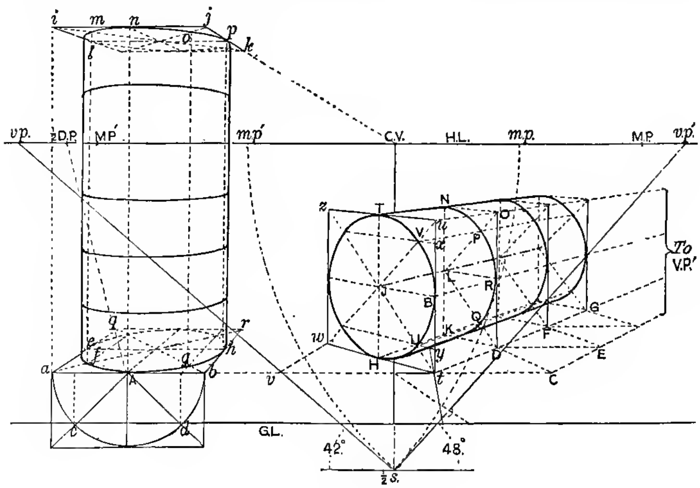


Fig. 71

about it describe a rectangle, as shown. Draw lines from A to the corners of the rectangle, cutting the semicircle in the points *c* and *d*. Draw perpendiculars from these points to meet *ab*, and join the meeting points to C.V. to determine the corresponding points *f* and *g*, in perspective. Draw a fair curve through these points, as shown. Draw *ai*, nine perspective feet in height, and draw a horizontal line from *i* to meet a vertical line at *b* in *j*. Join these points to C.V., and complete the square in perspective, with its diagonals, etc., as shown. Perpendiculars drawn from the points on the lower circle give corresponding points for drawing this upper circle. Divide each of these perpendiculars into six equal parts and draw fair curves through the points, as shown.

2. Find the necessary working points, and the point t (Prob. 13), which is to be the nearest bottom corner of a square circumscribing the circular end of the cylinder, as shown. Draw a perpendicular on t, and set off upon it $t\mathrm{B} = 2$ perspective feet, also Bu equal to tB. Join these three points, t, B, u, to each V.P. Set off $tv = 4$ perspective feet, and draw a line from v to M.P., meeting the receding line from t in w. Draw a perpendicular at w to meet the receding line from u in z. Then $twzu$ is a square in perspective. Draw diagonals to this square, also a vertical line through its centre.

As tu is equal in length to ab, set off ty, and ux, equal to the corresponding divisions on ab. Draw lines from x and y to V.P. These lines will give the points on the diagonals through which the circle forming one end of the cylinder will pass. Draw the circle, as shown.

Draw a horizontal line at t, and set off tC, three perspective feet in length. Draw a line from C to M.P.′ to meet the receding line from t in D. Draw a horizontal line from D to meet a line drawn from C to V.P.′ in E. Join E to M.P.′ to find F, and find G in a similar manner. Draw perpendiculars at the points D, F, and G to meet the receding line from u.

Draw lines from D, R, O, to V.P., meeting lines drawn from H, J, T, to V.P.′, in the points K, L, N. Join LD, LO, and draw lines from U, V, to V.P.′, meeting these lines in Q and P. Draw a fair curve through the points N, P, R, Q, K. Proceed in the same manner with the planes at F and G, and complete the cylinder, as shown.

Block letters are frequently given as subjects for perspective in the elementary stage of the science and art examinations. The following problem is an example.

Problem 29

Let A *and* A′ *represent the plan and elevation of the letter* P *cut out of a solid block. Represent the letter in perspective, on*

the same scale as the diagram, so that the corner B *may be on the* G.P., 3′ *from the* G.L., *and* 2′ *to the spectator's left, and the face of the letter in a vertical plane inclined to the right at an angle of* 50° *with the* P.P.

Distance of P.P. = 7′. *Height of eye* = 6′. *Scale* $\frac{1}{4} = 1'$.
Fig. 72.

Find the necessary working lines and points, and the point b (Prob. 12).

Join B′b, and produce the line to meet H.L. in v'. Draw a horizontal line through b.

About the two semicircles describe rectangles, with diagonals,

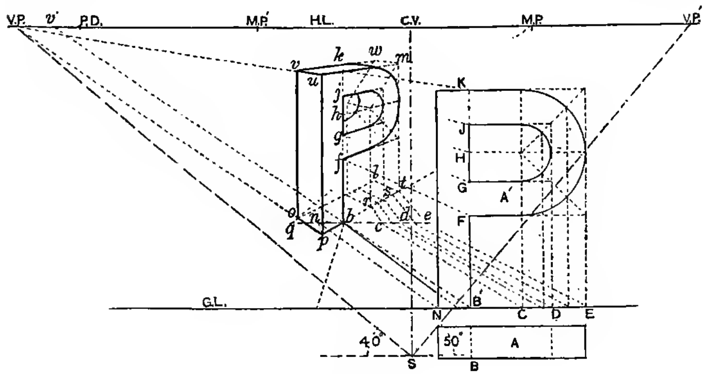


Fig. 72

etc., as in the preceding problem, and draw vertical lines from the several points to meet the G.L. in the points C, D, E, etc. Draw lines from these points to v' to meet the horizontal line through b in the points c, d, e, etc.

This point (v') is the V.P. for all lines parallel to B′b, and enables us to transfer the points C, D, E, to c, d, e, on a horizontal line through b.

Draw a line from V.P.′ through b, and a line from M.P.′ through n to meet it in p. Set off $nq = nb$, and draw a line from q to M.P. to meet a line from p to V.P in o.

Erect a perpendicular at b, and draw lines from F, G, H, etc., to v', to meet it in the points f, g, h, etc.

Draw lines from the points c, d, e, etc., to M.P.′, meeting the receding line from p in the points r, s, t, etc. Draw perpendiculars at these points to meet lines drawn from f, g, h, etc., to V.P.′, thus we shall get the necessary points for drawing the two semicircles on the face of the letter, as shown.

Draw a perpendicular at p to meet in u a line drawn from V.P.′ through k. Draw a perpendicular at o to meet in v a line from u to V.P.

The back of the letter is found by drawing lines from o and t to opposite V.P.'s to meet in l. Lines drawn from the points r, s, t, etc., to V.P., will give the necessary points on the line ol, at which to erect vertical lines to obtain the corresponding points on the back of the letter.

Concentric Circles

Problem 30

Draw three concentric circles in perspective, with diameters of 10′, 6′, *and* 3′, *respectively. The circles are to be in a plane perpendicular to both the* P.P. *and the* G.P.; *the nearest point* A *of the largest circle is to be* 4′ *from the* P.P., 12′ *to the spectator's left, and* 5′ *above the* G.P.

Distance of P.P. = 20′. *Height of eye* = 6′. *Scale* $\frac{1}{4}'' = 1'$. Fig. 73.

Find the necessary working lines and points (Prob. 5), the $\frac{1}{2}$ D.P. and the point a (Prob. 10).

Draw a perpendicular at a, and set off aA = 5 perspective feet, also Ae equal to aA. Join each of these points to C.V. Set off $ac = 5$ perspective feet, and draw c to $\frac{1}{2}$ D.P., meeting the receding line from a in d. Draw a perpendicular at d to meet the receding line from e in f. Then $adfe$ is the square circum-

scribing the largest circle. Draw diagonals to this square intersecting in *o*. Draw the perpendicular *st* through *o*, also the line A*m* towards C.V.

With any convenient point C as centre, and with a radius of 5′, draw a quadrant. Describe about this quadrant a square CEDB, as in the figure, and draw the diagonal DC. From the point F, where the quadrant cuts the diagonal, draw the perpendicular FG. Draw lines from the points B and G to C.V.

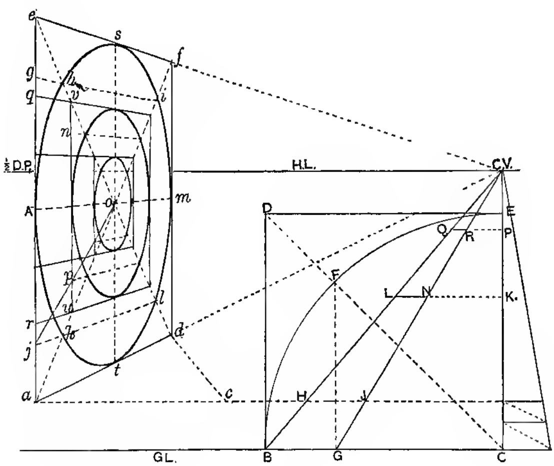


Fig. 73

Set off *aj* and *eg* each equal to HJ, and draw *j* and *g* to C.V. to meet the diagonals of the square in the points *k*, *l*, *h*, *i*. Draw a fair curve through the points A, *h*, *s*, *i*, *m*, *l*, *t*, *k*. This is the perspective of the largest circle.

Mark off *q* and *r*, each three perspective feet from A, and join them to C.V. The intersections of these lines with the diagonals will give the corners of the square circumscribing the second circle. Join these points to complete the square.

Join *jo*, cutting *uv* in *p*. Make *vn* equal to *up*, and draw lines from *n* and *p* to C.V. to meet the diagonals. Thus we shall determine the points through which the circle will pass. Deter-

mine the points for the smallest circle in the same manner, and complete the curves, as shown.

The points n and p could also be determined from the lines drawn from B and G to C.V., *e.g.* draw the horizontal line LK equal in length to half of vu, and set off vn and pu, each equal to LN. The points for finding the smallest circle could be determined in a similar manner from the line QP.

Problem 31

1. *Draw a right cone 5′ high, with base 4′ in diameter, standing on the* G.P., *the point* A *being 2′ from the* P.P. *and 4′ to the spectator's right.*
2. *Draw a cone of the same dimensions, lying on the* G.P. *with its axis parallel to the* P.P., *and the point* B *4′ from the* P.P. *and 3′ to the spectator's left.*

Distance of P.P. = 12′. *Height of eye* = 4′. *Scale* $\frac{1}{3}'' = 1'$. Fig. 74.

1. Find the necessary working lines and points (Prob. 5),

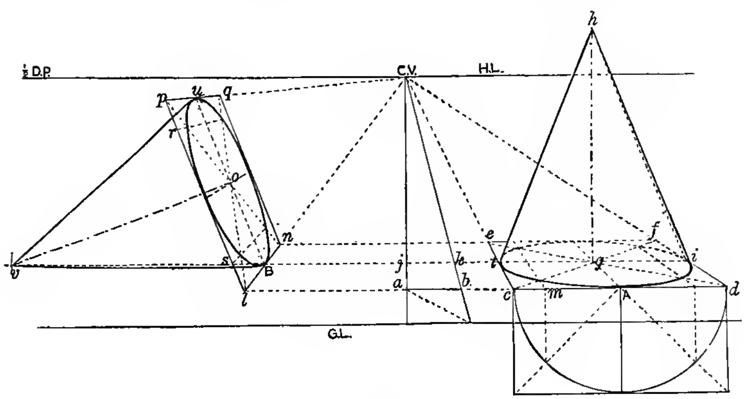


Fig. 74

the ½ D.P. and the point A (Prob. 10). Draw a horizontal line through A. Set off on each side of A the distances Ac and Ad, each equal to twice ab. Join these points to C.V. and complete

the square *edfe*. Draw the diagonals, etc., to this square, and determine the necessary points for drawing the circle in perspective (Prob. 25).

Draw a perpendicular at the centre *g*, five times *jk* in height, for the vertex *h*. Draw lines *hi*, and *ht*, tangents to the curve forming the base. This completes the cone.

2. Find the point B (Prob. 10). Draw a line from C.V. through B to meet the horizontal lines through *c* and *e* in *l* and *n*.

Draw a horizontal line through B, and make B*v* equal in length to *ih*. With B as centre, and radius equal to *ti*, draw an arc; and with *v* as centre, and radius equal to *v*B, draw another arc cutting the first in *u*. Join B*u*. Draw a line from C.V. through *u* to meet lines drawn from *l* and *n*, parallel to B*u*, in *p* and *q*. Then *lnqp* is the square circumscribing the base of the cone.

Draw diagonals to this square, also a line from C.V. through the centre. Set off *pr* and *ls*, each equal to *cm*, and draw lines from *r* and *s* to C.V. to cut the diagonals. Draw a fair curve, as shown, through the points found. This curve is the perspective of the circle forming the base of the cone, and we have only to draw lines *vu*, and *v*B, tangents to this circle to complete the cone.

Problem 32

Let A *and* B, A′ *and* B′ (Fig. 75) *represent to a scale of* $\frac{1}{4}'' = 1'$ *the plan and elevation of a rectangular solid supporting a cone. It is required to draw these solids in perspective to twice this scale, the longer edges of the solid* A *to incline to the left at an angle of* 40° *with the* P.P., *and the angle* C *to be* 3′ *from the* P.P. *and* 2′ *to the spectator's right.*
Distance of P.P. = 7′. *Height of eye* = 4′. Fig. 76.

Find the necessary working lines and points, and the point *c* (Prob. 13).

Produce a line from C.V. through *c* to meet the G.L. in C.

Draw the perpendicular C*a* twice the height of the solid A′. Draw a line from *a* to C.V., meeting a perpendicular on *c* in *b*.

Join *c* and *b* to each V.P. Draw a horizontal line through *c*. Set off CD twice the length of CD (Fig. 75). Draw a line from D to C.V., meeting the horizontal line through *c* in *d*. Draw a line from *d* to M.P., meeting the receding line from *c* in *f*. Find the opposite corner *g* of the same solid in a similar manner. Draw vertical lines at *f* and *g*, and complete the solid A, as shown.

Set off C*h* equal to twice CF (Fig. 75). Join *h* to C.V. to find *i*. Draw a line from M.P.′ through *i* to meet a line from V.P.′ through *c* in *j*. Set off C*r* = twice CG (Fig. 75), and find *k* in a similar manner to that described for *j*. Draw a line from V.P. through *j* to meet a line drawn from V.P.′ through *k* in *p*.

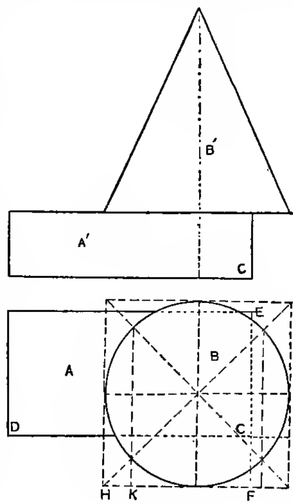


Fig. 75

Set off C*l* equal to twice FH (Fig. 75). Join *l* to C.V. to find *m*. Draw a line from *m* to M.P. to find *n*. Draw a line from V.P.′ through *n* to meet the receding line from *p* in *o*. Find the point *q* in a similar manner, and complete the perspective square *posq*. Draw diagonals to this square, intersecting in *t*.

Produce a line from V.P.′ through *p* to meet the G.L. in E. Draw the vertical line EF twice the height of A′ (Fig. 75). Draw a line from F to V.P.′ to meet vertical lines at *p* and *q* in *u* and *x*. Draw a line from *u* to V.P. to meet a vertical line at *o* in *w*. Draw lines from *w* and *x* to the V.P's. to complete the perspective square circumscribing the base of the cone. Draw diagonals to this square, and a line through the centre from each V.P.

To determine the points where a circle would cut the diagonals of this square, set off *l*K equal to twice HK (Fig. 75). Join K to C.V. to find N. Join N to M.P. to find L. Draw a line from V.P.′ through L to meet the diagonal in P. Draw a line from V.P. through P to meet the other diagonal, and thence

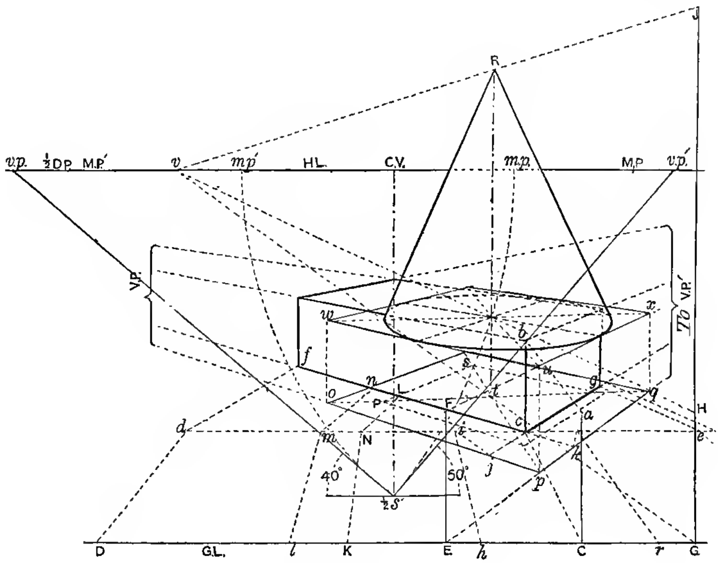


Fig. 76

a line to V.P.′ Draw perpendiculars at the points just found to the diagonals of the upper square, and draw a fair curve through them, touching the sides of the square, as shown.

Assume any convenient point *v* on H.L., and draw a line from it through *t* to meet the G.L. in G. Set up GH and HJ, equal respectively to twice A′ and twice B′ (Fig. 75). Draw J to *v*, cutting a perpendicular on *t* in R. Draw lines from R tangential to the base of the cone.

Curves

Problem 33

1. *The curve contained by the right angle* ABC *is drawn to a scale of* $\frac{1}{8}'' = 1'$. *Draw this curve to twice this scale in a vertical plane perpendicular to* P.P., *and resting so that the line corresponding to* BC *may be on the* G.P.; *the perspective of the point* B *is to be* 2′ *from the* P.P. *and* 4′ *to the spectator's left.*

Draw a part of the same curve on a plane parallel to and 2′ *to the left of the first plane, and join them so that the two curves may together form a thin opaque scroll.*

2. *Represent the same curve, lying to the left of the line* BC, *on the* G.P., *so that* BC *may be at right angles to the* P.P., *and the point* D 4′ *to the right of the spectator and* 2′ *from the* P.P.

Rotate this curve on the line BC *till the plane containing it is inclined to the* G.P. *at an angle of* 65° *on the right.*

Distance of P.P. = 16′. *Height of eye* = 6′. Fig. 77.

Assume any number of convenient points in the given curve,

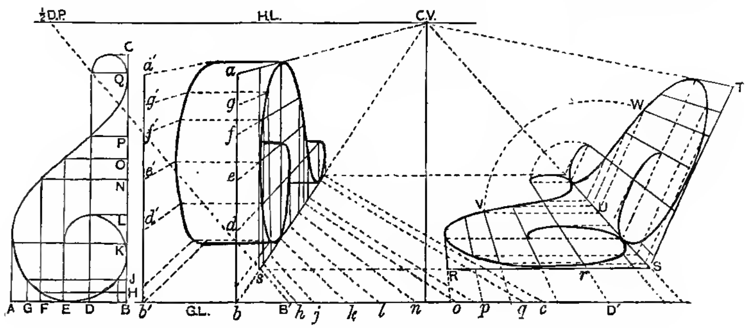


Fig. 77

and draw lines from them parallel to the lines AB and BC, as shown.

1. Find the necessary working lines and points (Prob. 5), the ½ D.P. and the point *s* corresponding to B (Prob. 10).

Draw a line from C.V. through *s* to meet the G.L. in *b*. Draw a perpendicular at *b*, and set up the heights *bd*, *be*, etc., equal to twice the corresponding heights B, BD, BE, etc. Join these points to C.V.

Draw a line from ½ D.P. through *s* to meet the G.L. in B′. Set off from B′, along the G.L., the distances B′*h*, B′*j*, B′*k*, etc., equal to the corresponding distances B, BH, BJ, BK, etc. Draw lines from *h*, *j*, *k*, etc., to ½ D.P. to meet the receding line from *b*, and draw vertical lines at these points to meet the receding lines from *d*, *e*, *f*, etc. Draw a fair curve through the intersections, as shown.

Set off *b*′, 2′ to the left of *b*, and draw a vertical line. Set off heights on this line equal to those on the vertical line at *b*. Draw lines from these points to C.V. to meet horizontal lines drawn from the corresponding points of the curve, and complete the scroll, as shown.

2. Draw a horizontal line through *s*. Set off D′, 4′ to the right of the spectator, and draw a line from D′ to C.V., intersecting the horizontal line from *s* in *r*. Mark off on the G.L., on each side of D′, points corresponding to the divisions on the perpendicular at *b*, and draw lines from them to C.V., intersecting horizontal lines drawn from the corresponding divisions on the receding line from *b*. Draw a fair curve through the corresponding intersections, as shown.

Draw the line ST at an angle of 65° with the G.P. on the right, and mark off upon it points corresponding to the divisions on RS, and draw lines from these points to C.V. Draw from the points on the receding line from S, lines parallel to ST. Draw a fair curve through the corresponding intersections, as shown.

This last curve might have been traced directly from the preceding as follows: Take any point, V for example, and draw a horizontal line through it to find U. With U as centre, and radius UV, draw an arc to meet the line UW, drawn parallel to ST in W. Then W will be a point on the new curve cor-

responding to V. Similarly any number of points on the new curve may be determined.

By the above example we have shown how to draw in perspective—

(1) *Parallel curves in vertical planes;* (2) *a curve in a horizontal plane;* (3) *a curve in an inclined plane;* (4) *a plane curve rotated through a given angle about a line in its own plane.*

When a curve is of a more complex character, it is better to intersect it with a series of parallel lines at equal distances apart, and at right angles to each other, forming small squares. These squares can then be drawn in perspective, and the curve drawn by eye through the corresponding squares. This method is illustrated in the following problem.

PROBLEM 34

The curves enclosed in the square ABCD *are drawn to a scale of* $\frac{1}{6}'' = 1'$. *Draw them to twice this scale, lying on the* G.P. *with the point* E 2′ *from the* P.P. *and directly opposite the spectator.*

Distance of P.P. $= 8\frac{1}{2}'$. *Height of eye* $= 5'$. Fig. 78.

Divide each side of the square into any number (7) of equal

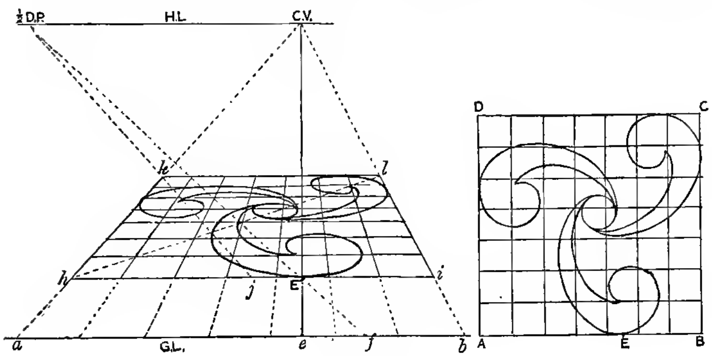


Fig. 78

parts, and draw lines from the points of division parallel to the sides.

Draw the G.L. and H.L.

In any convenient position on the G.L. set off *ab* equal to twice AB, and divide it into seven equal parts. Take *e*, the middle point of the third division from the right, as the point corresponding to E, and erect a perpendicular at *e* to find the C.V. Draw lines from each of the points on the G.L. to C.V.

Find the $\frac{1}{2}$ D.P. (Prob. 10). Set off $ef = 1'$, and join *f* to the $\frac{1}{2}$ D.P. to find E′. Draw a horizontal line *hi* through E′. Bisect *hi* in *j*, and draw a line from *j* to $\frac{1}{2}$ D.P. to find *k*. Draw through *k* the horizontal line *kl* and join *hl*. This line is a diagonal of the perspective square *hilk*. Draw horizontal lines through each of the points where this diagonal intersects the receding lines from the points on the G.L.

Draw by the eye through the corresponding squares a curve, as shown, corresponding to the given figure.

Problem 35

Let A *be the elevation of a large circular bowl drawn to a scale of* $\frac{1}{6}'' = 1'$. *Draw it to twice this scale in perspective, with its axis* 4′ *from the* P.P. *and* 5′ *to the spectator's left.*
Distance of P.P. = 12′. *Height of eye* = 6′. Fig. 79.

Find the necessary working lines and points (Prob. 5), the $\frac{1}{2}$ D.P. and the point *c* (Prob. 10).

Draw a line from C.V. through *c* to meet the G.L. in C′. Set off from C′ the distances C′D′, C′E′, C′B′, equal to twice the corresponding distances CD, CE, CB. Draw lines from the points D′, E′, B′ to C.V., meeting a horizontal line through *c* in the points *d*, *e*, and *b*.

Draw a perpendicular on C′, and mark off the points F′, G′, etc., at twice the heights of the corresponding points F, G, etc. Draw lines from these points to C.V. to meet a perpendicular on *c* in the points *f*, *g*, etc. Draw horizontal lines from *f*, *g*, etc., to meet perpendiculars at the points *d*, *e*, and *b*, in the points *k*, *l*, *m*, and *n*. Produce the horizontal lines and set off equal

distances on the opposite side of the axis to find the corresponding points *o*, *p*, *q*, etc. Draw lines from P.D. through these points to meet receding lines from the points C′, F′, etc., in the points *w*, *v*, *u*, etc.

We will now find the M.P. accompanying the P.D. The distance of *s* from C.V. is equal to half the distance of P.P.; therefore with $\frac{1}{2}$ D.P. as centre, and the distance of *s* from

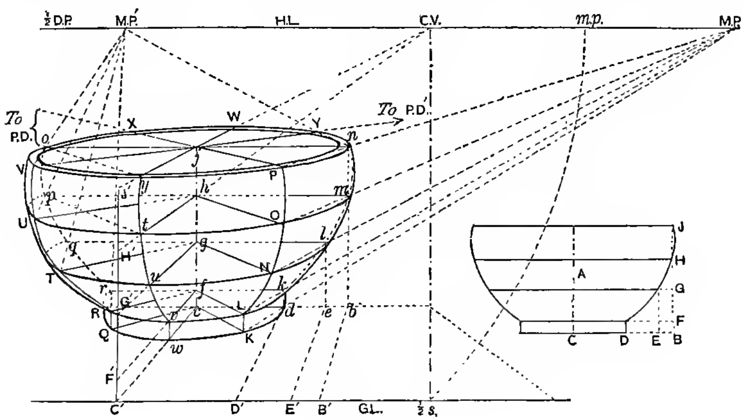


Fig. 79

$\frac{1}{2}$ D.P. as radius, describe an arc to meet the H.L. in *m.p.* At twice the distance of *m.p.* from C.V. we find the M.P. required.

Draw lines from P.D. through the points *j*, *h*, *g*, etc., to meet lines drawn from M.P. through the points *n*, *m*, *l*, etc., in the points P, O, N, etc.

Find P.D.′ [23] and set off M.P.′ the same distance from C.V. as M.P. Draw lines from P.D.′ through the points *j*, *h*, *g*, etc., to meet lines drawn from the points *o*, *p*, *q*, etc., in the points V, U, T, etc.

The more distant points of the rim are found by drawing *o* to M.P. to meet a line from *j* to P.D. in X. A line from *n* to P.D. meets *yj* produced in W; and a line from *n* to M.P.′ meets a line from *j* to P.D.′ in Y.

Draw fair curves, as shown, through the points found.

This problem may serve to illustrate curves drawn on vertical planes through a common axis, if the extremities of the parallel lines on each plane are joined, as shown. The problem might also have been worked by finding, in perspective, horizontal circles at the different levels, and drawing curves, tangentially, to them.

EXERCISES

PRACTICE

The scale to be used in working all these problems is half an inch to 1 foot.

The eye is supposed to be, in each case, 13′ distant from the P.P., *and* $5\frac{1}{2}$′ *above the* G.P.

1. Draw in perspective a rectangular slab of dimensions 6′, 6′, and 1′ respectively, resting with one of its narrow faces on the G.P., at right angles to the P.P.; its nearest edge being 1′ from the picture and 5′ to the spectator's left. Represent a circular hole 4′ in diameter cut through the centre of the slab.

2. Draw in perspective a circle 5′ in diameter, lying on the G.P. with its centre 7′ from the G.L. and 3′ to the right of the spectator. Let the circle form the base of a cylinder 2′ high.

3. Draw in perspective a square of 4′ side at right angles to the P.P. and inclined to the G.P. at an angle of 30° to the right, having one of its sides on the G.P. and its nearer lower corner 2′ from the P.P. and 2′ to the spectator's left.

Within the square inscribe a circle 4′ in diameter, and let it form the lower end of a cylinder 1′ thick.

4. The cone illustrated in Fig. 74 has a base 4′ in diameter and an axis 5′ long. Draw a cone in a similar position, with reference to the picture and ground planes, to that represented in the left-hand figure, but with the point B 6′ to spectator's right.

5. Draw three concentric circles with diameters of 5′, 4′, and 2′ respectively, in a plane perpendicular to the P.P., and inclined to the G.P. at an angle of 30° on the left; so that the lowest point of the largest circle may touch the G.P. 4′ from the G.L. and 1′ to the spectator's left.

6. Draw a hollow cylinder 4′ in external diameter, 4′ in length, and 6″ in thickness. Let it lie on the G.P. with its axis horizontal and inclined to the P.P. at an angle of 40° on the right, the nearer end of its axis being 2′ from the P.P. and 1′ to spectator's left.

7. Draw a curve similar to that illustrated in Fig. 77, with the line AB 3′ long. Let this line be on the G.P., perpendicular to the G.L., and let A, the nearer end of the line, be 2′ from the G.L., and 2′ to the spectator's left, and let the plane containing the curve be inclined to the G.P. at an angle of 30° on the left.

Draw in perspective the plane ABCD, as in Fig. 78, and with similar curves thereon. Let the line AB be 6′ long, and recede to the left at an angle of 45° with the P.P., and let the point B, the nearer end of the line, be 3′ from the G.L. and 2′ to spectator's left.

Theory

1. Through what point on the horizon line does the vanishing line for planes perpendicular to the picture and ground planes pass [34]?

2. Why is the angle of vision limited to 60°? How would the representation of an object be affected if this limit were extended [45]?

3. Why should lines perpendicular to the picture converge to the centre of vision [17]?

4. Explain how measuring points enable us to measure distances on receding lines [29]?

5. If a plane were perpendicular to the picture plane, and inclined to the ground plane at an angle of 45°, through what point on the horizon line would its vanishing line pass [34]?

6. Where is the vanishing line for planes at right angles to the ground plane, and forming an angle of 45° with the picture plane [36]?

CHAPTER VI

MODELS FOR PRACTICE

THE illustrations in this chapter are rather more elaborate in character than those generally given in the elementary stage, but they form good subjects for the practical application of several of the problems already given.

Fig. 80 represents in parallel perspective a model built up of blocks, the majority of the dimensions being one foot and half a foot.

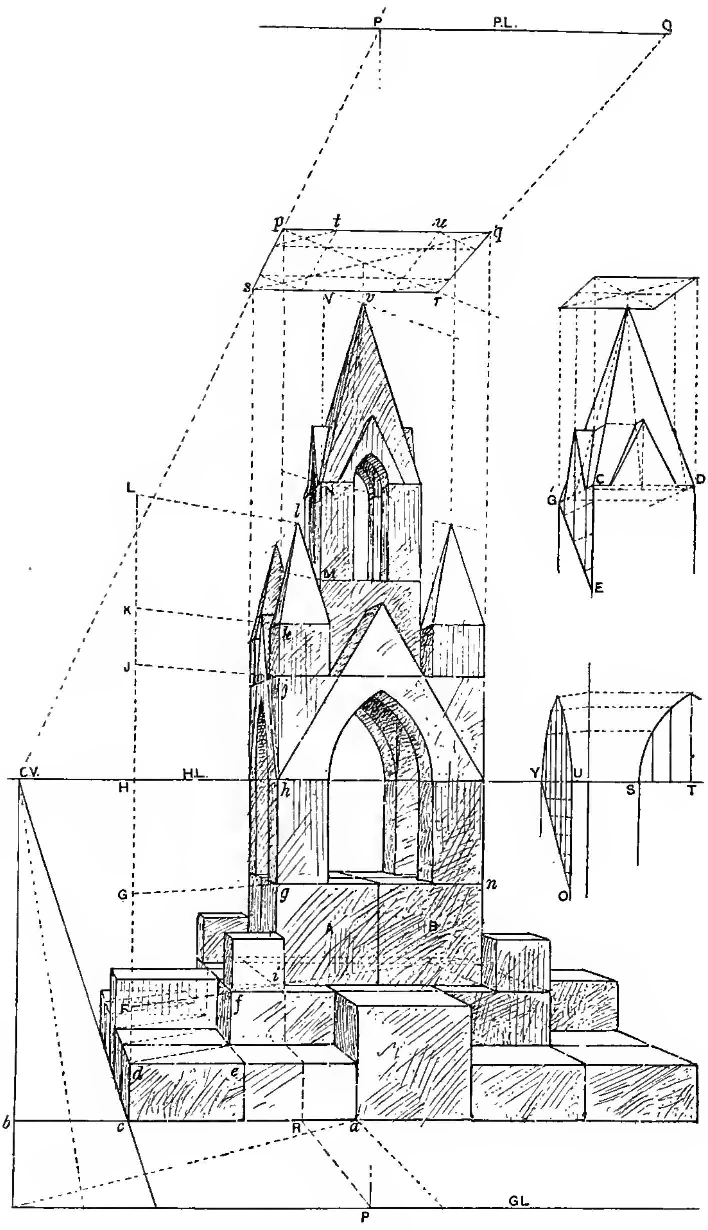
The point a is 3′ from the P.P. and 3′ to the spectator's right. *Distance of* P.P. = 12′. *Height of eye* = 3′. *Scale* $\frac{3}{4}''$ = 1′.

Find the necessary working lines and points, perspective scale, and point a (Prob. 5). As several of the dimensions are half a foot, bisect the perspective scale, as shown.

Draw a horizontal line through a and measure, as in previous exercises, the widths of the five front blocks, as well as the height of the middle one, with the perspective foot bc. The height of the other blocks is half of bc.

Join the top corners d and e of the left-hand block to C.V., and measure five receding feet, as shown, by means of the P.D.

The student should be able to determine the position of the successive tiers of blocks from the construction shown, *i.e.* by erecting a perpendicular at d, and setting off upon it the points F, G, H, etc., at intervals of feet and half-feet, as required, using the perspective foot bc. Lines drawn from these points to P.D. give the corresponding points f, g, h, etc.


P
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F
f
d
e
b
c
R
a
P
G.L.
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O

Fig. 80

The tops of the blocks A, B, etc., are too narrow to allow us to construct with accuracy the receding widths of the blocks resting upon them. In such a case it is advisable to construct what is called a *floating plane*, which we do in the following manner:—

Produce the perpendiculars at *g* and *n* to any convenient height, and join them by the horizontal line *pq*. Join *p* and *q* to C.V., and draw a line from *p* to P.D. to meet the receding line from *q* in *r*. Draw the horizontal line *rs*. Then *pqrs* forms a floating plane, *i.e.* a horizontal plane suspended in mid-air. This plane could have been constructed below the G.P. if such a position had been more convenient.

The picture line [40] for this plane may be determined by finding first where a vertical plane containing the line *ps* will meet the P.P.; such a plane will intersect the G.P. in the line RP, the construction for which is shown, and the G.L. in P. Let a perpendicular be drawn from P to meet *sp* produced in P′; then the line PP′ is the intersection of the vertical plane with the P.P.; hence the horizontal line P′Q is the picture line required.

Set off *pt* and *qu*, each equal to half of *gh*. Join them to C.V., and where they meet the diagonal *pr*, draw horizontal lines, as shown; by drawing perpendiculars from the several points obtained, we determine the receding widths of the blocks required.

The positions of the vertices of the pyramids are found by drawing the diagonals of the perspective squares in the floating plane, and letting fall perpendiculars from their centres, as shown at *v* for example. To find the height of this point, draw the perpendicular NV equal to twice MN, and draw a line from V to P.D.

The construction for finding the small gables at the top of the model is shown in the upper supplementary drawing. First draw the gable in front to the dimensions shown. Mark off divisions on CE equal to those on CD. Join EG′. Draw lines

from the points on CE to C.V. to meet EG′, and erect perpendiculars, as shown.

The lower supplementary drawing shows how a Gothic arch is drawn in perspective; the same principle could be applied to arches of any form.

First draw half of the front arch geometrically. Divide ST into any number of equal parts, three for instance, and draw perpendiculars from them to meet the arch. Draw horizontal lines from these points.

Determine the points U and Y by means of a floating plane, as already described.

Set off on UO six equal divisions, any size, and join OY. Draw lines from the divisions on UO to C.V. to meet this line, and then erect perpendiculars to meet the horizontal lines drawn to C.V., as shown.

Fig. 81 represents, in angular perspective, a model whose front forms an angle of 40° on the right with the P.P. It is built of blocks, the principal dimensions of which are 1 foot and 2 feet.

This model shows how the receding widths of the blocks may be measured without using the ordinary measuring points. The point a is 2′ from the P.P., and 2′ to the spectator's left.

Distance of P.P. $= 15'$. *Height of eye* $= 4\frac{1}{2}'$. *Scale* $\frac{1}{3}'' = 1'$.

Determine, by means of a $\frac{1}{3}$ station point (Prob. 13), the $\frac{1}{3}$ D.P., V.P.'s, and point a.

In any convenient position on the G.L., take b and draw bd and bc, each 1 foot in length, at the required angles with the G.L. Draw lines from c and d perpendicular to the G.L. Then bf is the *picture size length of a foot* (*i.e.* the length in the P.P.) receding to the right, and be of one receding to the left.

Produce a line from C.V. through a to meet the G.L. in g. Set off $gh = eb$, and draw a line from h to C.V., meeting a line from V.P. through a in H. Join H to V.P.′ Set off $hj = bf$, and draw a line from j to C.V. to meet the receding line from H in J. Find all the other receding widths in a similar manner.

The smaller arches are 2′ wide, and the larger arch 4′ wide. The widths on the extreme right can be found without going beyond the limits of our picture, as follows :—

Having drawn lines from *m* and *n* to C.V., draw any horizontal line *or*. Set off *op* the required number of times on

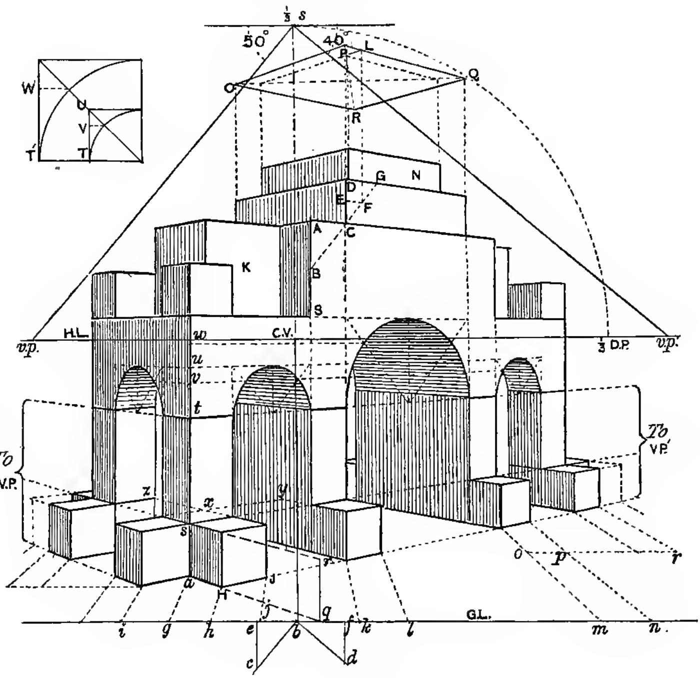


Fig. 81

this line and draw lines from the points thus found to C.V., meeting the receding line from H. Find the widths on the left-hand side of the model in a similar manner.

Draw a perpendicular at *a*, and find the height *as* from the line of heights *qr* [48]. Set off the required dimensions on *as* produced, and join the points thus found to the V.P.'s.

In a supplementary drawing with radii equal to *as* and *st* respectively, describe two concentric quadrants, and determine where the diagonal of the square containing them will cut

the arcs, as shown. Draw the necessary lines for constructing the arches in perspective (Prob. 27), and set off $tv = \mathrm{TV}$, and $tw = \mathrm{T'W}$. Draw lines from v and w to V.P.′ and V.P., to meet the diagonals, and draw the arches, as shown.

The position of block K is determined by drawing perpendiculars from the points z, x, and y.

Draw the square OPQR in a floating plane, at any convenient height, and draw diagonals to it. See Fig. 80.

There is a horizontal margin of half a foot surrounding the uppermost block N ; this could be found as follows: Bisect AS in B. Join BC and produce the line to G. Bisect DC in E, and draw a line from E to V.P.′ to meet CG in F. As DG is one foot, EF is equal to half a foot. Draw a perpendicular from F to meet the floating plane in L. Draw a line from L to V.P. to meet the diagonals of the floating square. Complete the inner square, as shown. Draw perpendiculars from the corners of this square and complete the block N.

Fig. 82 shows how the perspective position of a model can, if necessary, be drawn without using the G.P.

The block A is $5' \times 5' \times 1'$, B is $4' \times 4' \times 1'$, and the cylindrical blocks are $4'$ in diameter and $16''$ in length.

The point a, the middle point of of, is $2'$ from the P.P., $4'$ to the spectator's right, and $5'$ above the level of the eye.

Distance of P.P. $= 12'$. *Scale* $\frac{1}{2}'' = 1'$.

Find the H.L., C.V., and $\frac{1}{2}$ D.P. (Prob. 10).

Draw a perpendicular at C.V. and find b, $5'$ above H.L. Draw the horizontal line bc, $4'$ long. Set off $cd = 1'$, and draw a line from d to $\frac{1}{2}$ D.P. to meet a line from c, drawn to C.V., in a. Draw a horizontal line through a, and a line from d to C.V., meeting it in e. Set off af and ao, each equal to twice ae. Make $fg = ae$, and complete the front of B.

Bisect lg in h, and draw a line from h to $\frac{1}{2}$ D.P. to meet a line from g to C.V. in j. Complete the underside of B, and draw the diagonals, etc., as shown.

With radius equal to hl, draw a quadrant as a supplementary

drawing. Construct the square with diagonal, etc., as shown, and find N. Set off *gk* and *ln*, each equal to LN, and draw lines from *k* and *n* to C.V., meeting the diagonals, as shown, and draw the circle in perspective (Prob. 25).

Draw perpendiculars from each point marked on the circle to meet the H.L., and divide each perpendicular into three

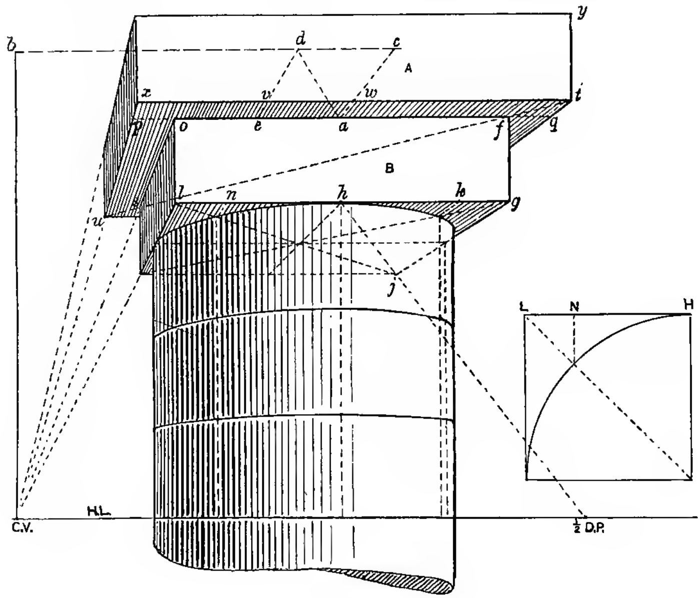


Fig. 82

equal parts. Draw fair curves through the points thus found, as shown.

The block A projects half a foot beyond the block B. Produce *of*, and set off *fq* and *op*, each equal to half of *fg*, and draw lines from C.V. through *p* and *q*. Draw the diagonal *sf*, and produce it to meet the lines through *p* and *q* in *u* and *t*. Draw horizontal lines from *t* and *u* to complete the underside of block A. Draw perpendiculars at the front corners, *x* and *t*, each equal to *vw*, and complete the block, as shown.

Fig. 83 represents a doorway with steps. The principal

dimensions of the blocks are 2′, 1′, and $\frac{1}{2}$′; the carpet is 2′ wide. The working points are found by means of $\frac{1}{3}$ station point (Prob. 13). The horizontal lines recede at angles with the P.P. of 50° on the right and 40° on the left. The point *a* is 3′ from the P.P. and 2′ to the spectator's left.

Distance of P.P. = 9′. *Height of eye* = 4′. *Scale* $\frac{1}{2}$″ = 1′.

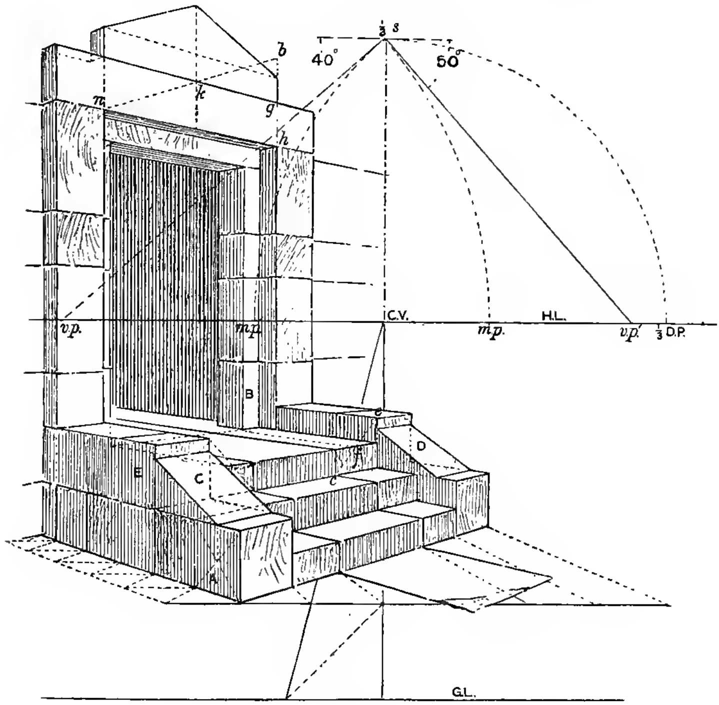


Fig. 83

This model is a development of Prob. 13, and two methods of finding the receding widths have been shown, as explained in that problem.

The edges of the sloping surfaces C and D are found as follows: Let C intersect block A half a foot from its front, and block E a quarter of its height from the top, and their edges may easily be drawn. If these edges were produced they would,

since they all lie in vertical planes parallel to one side of the model, meet in an A.V.P. [39], on the perpendicular drawn at the V.P.

The block B is half a foot wide. This width could be determined as follows: Join *ce* by a line cutting the top of the step in *f*, and join *f* to V.P.

To find the centre *k* at the top of the doorway, make $gb = gh$, and join *nb*.

EXERCISES

Practice

1. Draw the model (Fig. 81) in parallel perspective, using a half-distance point (Prob. 10). Its shorter side to be parallel to the P.P., and the point H to be 3′ from the G.L. and 4′ to spectator's left.
Distance of P.P. = 16′. *Height of eye* = 4′. *Scale* $\frac{3}{4}'' = 1'$.

2. Draw the model (Fig. 82) in angular perspective. The line *lg* is to recede to the right at an angle of 36° with the P.P., and the point *l* is to be 4′ above the level of the eye, 3′ from the P.P., and 2′ to the spectator's left.
Distance of P.P. = 12′. *Scale* $\frac{1}{2}'' = 1'$.

3. Draw the model (Fig. 83) in parallel perspective, using a half-distance point (Prob. 10). The edges of the steps are to be parallel to the picture. The point *a* is to be 3′ from the G.L. and 2′ to spectator's right.
Distance of P.P. = 14′. *Height of eye* = 5′. *Scale* 1″ = 1′.

4. Draw the model (Fig. 80) in angular perspective, using a half-station point (Prob. 13). The right-hand side of the model is to recede to the right at an angle of 40° with the P.P. The point *c* is to be 2′ from the G.L. and 3′ to spectator's left.
Distance of P.P. = 12′. *Height of eye* = 5′. *Scale* $\frac{3}{4}'' = 1'$.

Theory

1. What is a floating plane? Explain its use (p. 108).

2. What do you understand by the following phrase—"The picture length of a receding foot" (p. 109)?

3. Why do the perspective dimensions of objects outside the field of vision appear exaggerated? Explain by means of a diagram (Fig. 34).

CHAPTER VII

THE DIRECT METHOD OF PERSPECTIVE

THIS method consists in finding a series of points in perspective *directly* from their known positions. The object must be first drawn, geometrically, in plan to show its relative position with reference to the spectator and the picture plane; we can then

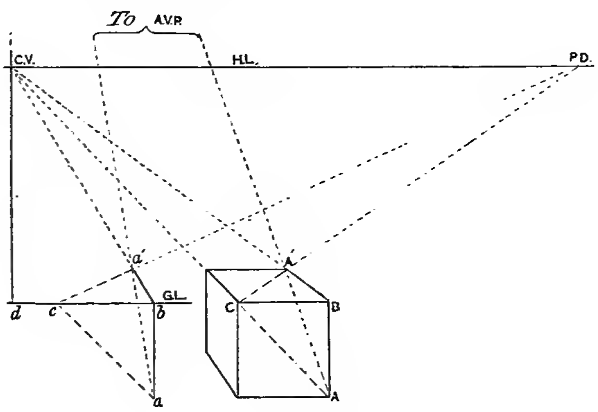


Fig. 84

select suitable points and find their corresponding positions in perspective. By joining these points in accordance with the plan we obtain a perspective view of the object.

If the object we wish to draw has an elevation, we can determine the perspective heights of the several points according to their respective distances, and by joining them in accordance

with the corresponding points in the elevation, we can obtain a perspective view of the object.

First let us consider a point on the G.P., having therefore no elevation.

Let a (Fig. 84) represent the plan of a point on the G.P., db its distance to the spectator's right, and ba its distance from the P.P.; also let the distance of P.P. $= 12'$, the height of the eye $5'$, and the scale $\frac{1}{4}'' = 1'$.

If we assume db to be the G.L., $5'$ above it will give us the position of the H.L.; a perpendicular on d will determine the C.V., and $12'$ from C.V. along H.L., the P.D.

Make $bc = ba$. Join b to C.V. and c to P.D. The intersection of these lines gives us a', the perspective of a.

We shall understand the relative positions of these two points better if we draw a cube in parallel perspective with its edge CB at the same level, and equal in length to cb. As the top of the cube is equal to its front, the triangle represented by BCA′ must be equal to the triangle BCA, and the position of the point, of which A′ is the perspective, must be the same distance from B as the point A.

If we were to join aa', and produce the line to meet a perpendicular at C.V., their intersection would be at the same distance from C.V. as the P.D. is, *i.e.* the distance of spectator from P.P. [23]. This point is the A.V.P. for all lines parallel to aa', as AA′; so we could find a' without using the point c or the P.D. by means of this A.V.P. This method is even more direct than that already described, although the former method is the one generally used.

Problem 36

To draw in perspective a rectangle on the G.P., *its plan being given. Distance of* P.P. $= 10'$. *Height of eye* $= 5'$. *Scale* $\frac{1}{4}'' = 1'$. Fig. 85.

Let ABCD be the plan of the rectangle. Draw lines from each corner perpendicular to the G.L., and meeting it in the points A′B′D′C′. With A′ as centre and radius A′A, describe an arc to meet G.L. in a. Draw lines from A′ to C.V., and from a to

P.D., meeting in a'. Find the other points in a similar manner and join them, as shown. Then $a'b'c'd'$ is the rectangle in perspective.

If we were to produce the parallel sides of this rectangle we

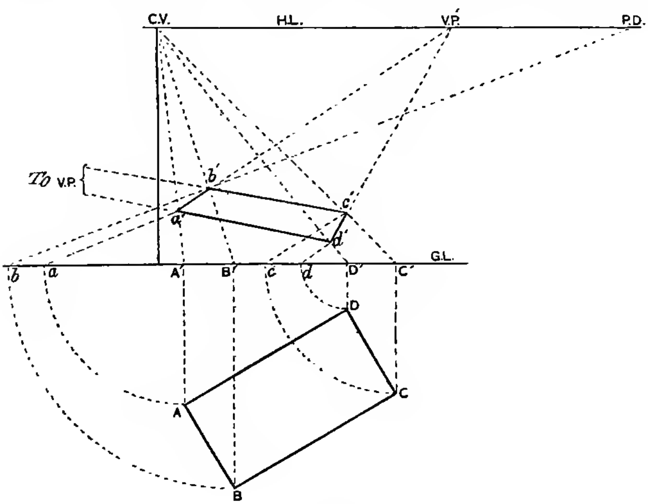


Fig. 85

should find, since they are receding *horizontal* lines [20], that they would meet in V.P.'s on the H.L.

If we were to make a drawing of Fig. 85, and crease the paper at the G.L., so that the plan would lie on a horizontal plane, and the perspective drawing on a vertical plane, as shown in Fig. 86, stretched threads running from the points on plan through the corresponding points in perspective would all meet at a point E, the position of the eye. The threads would represent visual rays [9], the horizontal plane the ground plane [2], and the vertical plane the picture plane [3].

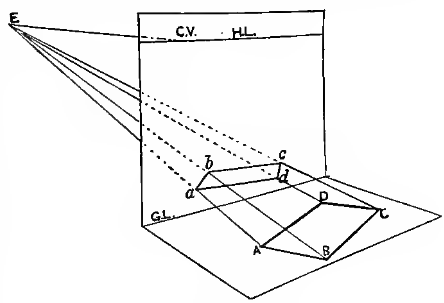


Fig. 86

PROBLEM 37

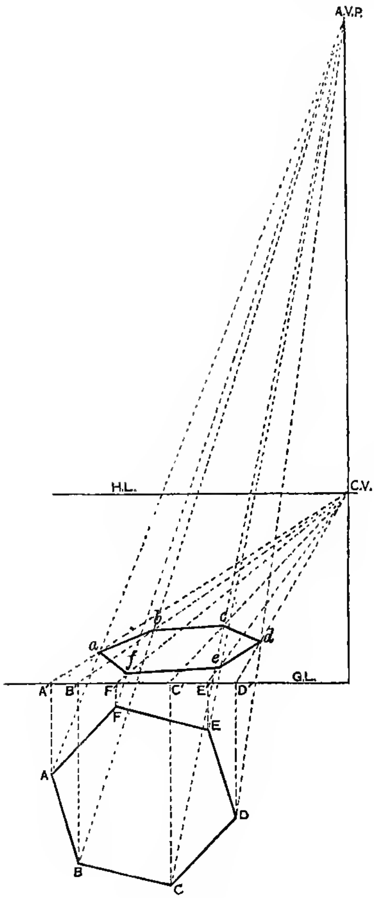


Fig. 87

To draw a regular hexagon in perspective without using the P.D., *its plan being given in position.*

Distance of P.P. = 10′. *Scale* $\frac{1}{4}'' = 1'$. *Height of eye* = 4′. Fig. 87.

Let ABCDEF be the plan of the hexagon which is supposed to be on the G.P. Draw a perpendicular at C.V. equal in length to the distance of spectator from P.P., to find the A.V.P.

Draw from the corners of the hexagon lines perpendicular to the G.L. aud meeting it in the points A′B′F′C′E′D′. Draw lines from these points to C.V. Lines drawn from the points in plan to A.V.P. will intersect the receding lines from A′, B′, etc., in the points *a*,*b*,*c*, *d*,*e*,*f*. Join these points, as shown.

PROBLEM 38

To determine the true shape of an irregular plane figure from its perspective representation.

Distance of P.P. = 10′. *Height of eye* = 6′. *Scale* $\frac{1}{4}'' = 1'$. Fig. 88.

Let A′B′C′D′E′F′ be the perspective view of the figure.

Draw lines from C.V. through these points to meet the G.L. in the points *c, e, d, f, a*. The points B′ and E′ happen to be on the same line. Draw lines from P.D. through the same points to meet the G.L. in the points *a′b′f′d′c′e′*.

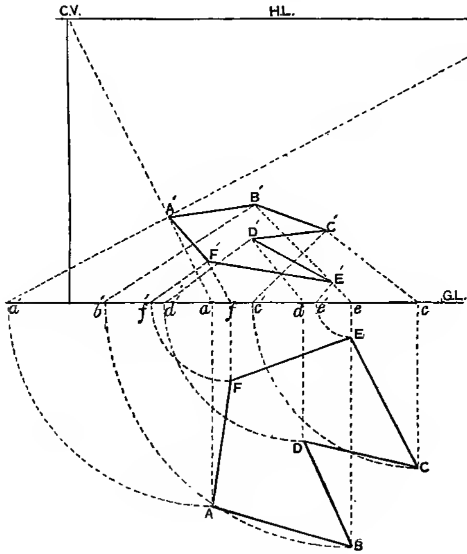


Fig. 88

With *a* as centre, and radius *aa′*, describe an arc to meet a perpendicular from *a* in A. Find the other points in a similar manner, and join them, as shown.

Problem 39

Two rectangular solids, A *and* B, *each* 6′ × 2′ × 1′, *being given in position on plan, it is required to draw the same in perspective.*

Distance of P.P. = 11′. *Height of eye* = 5′. *Scale* $\frac{1}{4}''$ = 1′. Fig. 89.

Find the positions of the points *c, d, h, g, e, f, k, j* on the ground plane by the method described in Prob. 36.

Draw a perspective scale for 1 foot (Prob. 5).

To determine the height of the solid A′ at *f*, draw a horizontal line from *f* through the perspective scale to obtain the perspective foot *ln*. Draw the perpendicular *fo* equal to *ln*.

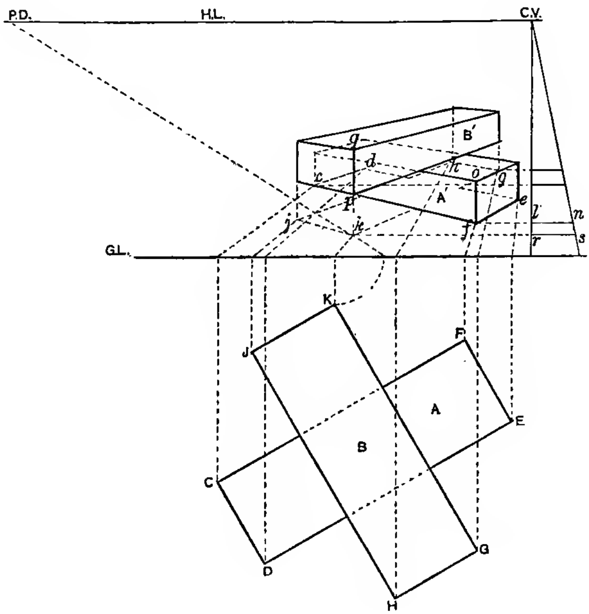


Fig. 89

Determine the heights of the other vertical edges of A′ in a similar manner, and complete the solid.

To determine the height of the solid B′, draw a horizontal line from *k* through the perspective scale to find *rs*. Draw a perpendicular at *k*, and set off *kp* and *pq*, each equal to *rs*. Find the other vertical edges of B′ in a similar manner, and complete the solid.

PROBLEM 40

To draw a cylinder in perspective 2′ in height, its plan being given in position.

Distance of P.P. = 12′. *Height of eye* = 6′. *Scale* $\frac{1}{4}''$ = 1′. Fig. 90.

Let ADBE be the plan of the cylinder.

Draw the diameter DE at right angles to the G.L., and produce it to meet the G.L. in K. Join K to C.V. and find the points *e*, *c*, and *d* (Prob. 36).

Draw the diameter AB parallel to the G.L. and a horizontal

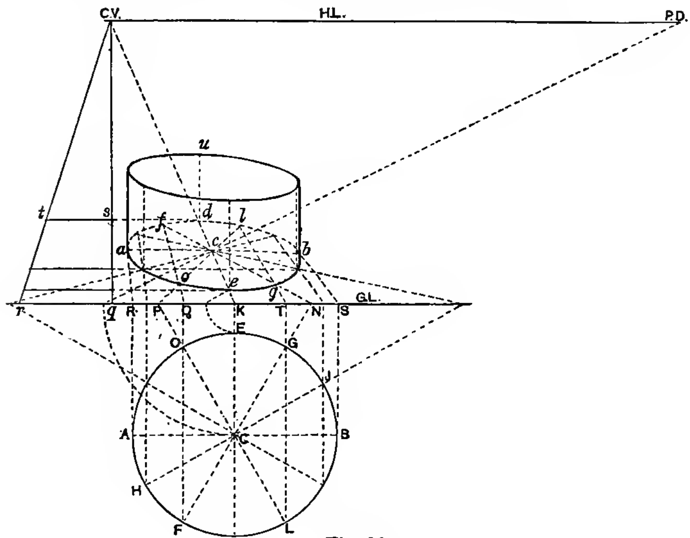


Fig. 90

line through *c*. Draw perpendiculars at A and B to meet the G.L. in R and S. Draw lines from R and S to C.V. to meet the horizontal line through *c* in *a* and *b*.

Draw any number of diameters to the circle which forms the plan of the cylinder, and produce them to meet the G.L. Take two of them, *e.g.* the diameters FG and LO, equally inclined to ED and meeting the G.L. in the points N and P. Draw lines from N and P through *c*. Draw perpendiculars through FO and LG to meet the G.L. in the points Q and T. Draw lines from Q and T to C.V., meeting the lines from N and P in the points *g*, *f*, *o*, *l*. Determine the other points of the

circle on the G.P. in a similar manner, and draw a fair curve through them.

To determine the height of the cylinder, draw a perspective scale for 2′ by setting off *qr* on G.L. = 2′, and drawing *r* to C.V. Draw perpendiculars at the points found of the perspective circle, and determine their proper heights, *e.g.* to find the height of the perpendicular at *d*, draw a horizontal line from *d* through the perspective scale to find *ts*, and make *du* equal to *ts*. Find the other heights in a similar manner, and draw a fair curve through them. Complete the cylinder, as shown.

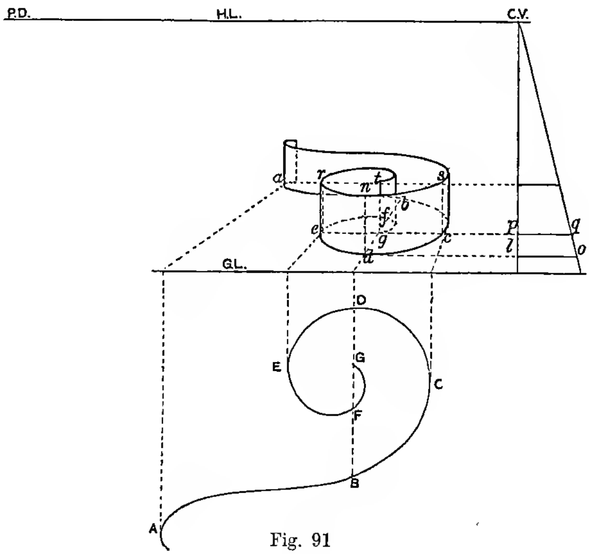


Fig. 91

Problem 41

To draw in perspective, from a given plan, a scroll 1′ *in height, lying upon the* G.P., *with its faces vertical.*

Distance from P.P. = 8′. *Height of eye* = 4′. *Scale* $\frac{1}{3}'' = 1'$. Fig. 91.

Select any number of convenient points in the curve. We

take only a few in the illustration to avoid confusion, viz. A, B, C, D, E, F, G. Find the perspective positions of these points on the G.P. (Prob. 36), and draw a fair curve through them.

Construct a perspective scale for 1′ (Prob. 5).

To determine the height of the scroll, it is not necessary that the points we have already found should be used. It is more convenient to take as many heights as we can at the same distance, *e.g.* draw a horizontal line through *g;* this will meet the curve in three points, viz. *e*, *g*, and another point near *c*. The perspective foot *pq* will give the height of the perpendiculars at these points. The perpendicular *dn* is equal to the perspective foot *lo*.

Find other points in a similar manner, draw a fair curve through them, and complete the scroll, as shown.

EXERCISES

Practice

The scale to be used in working all these problems is ½ inch to 1 foot.

The eye is supposed to be, in each case, 14′ *distant from the* P.P., *and* 5′ *above the* G.P.

1. Find the position, in perspective, of a triangle formed by three points, A, B, and C. A is to be 2′ from the G.L. and 3′ to spectator's left; B is to be 4′ from the G.L. and 2′ to spectator's right; C is to be 6′ from the G.L. and 5′ to spectator's left. Determine from the perspective drawing a plan of the triangle.

2. Draw a square of 4′ side lying on the G.P., with its nearer side receding to the right at an angle of 30°; its nearest corner being 1′ from the G.L. and 2′ to spectator's left. Determine from its perspective position the distance of the centre of the square from the G.L.

3. Draw the regular hexagon (Fig. 87) as a plan, with the point F 1′ from the G.L. and 4′ to spectator's left, and the side FE forming an angle of 30° with the G.L. Represent the hexagon in perspective lying on the G.P., and let it be the base of a pyramid, the axis of which is 6′ high. Complete the pyramid.

Note.—The sides of the hexagon (Fig. 87) are 2′ long.

4. Draw the rectangle (Fig. 85) as a plan, with the point D 2′ from the G.L. and 5′ to spectator's right, and the line DA making an angle of 40° with the G.L. Draw the rectangle in perspective lying on the G.P., and let it represent one side of an equilateral triangular prism. Complete the prism.

Note.—The rectangle (Fig. 85) is 4′ × 2′.

Theory

Note.—These questions refer to the "direct" method of perspective only.

1. What is the direct method of perspective (p. 115)?
2. What relation has the plan of the object, drawn below the G.L., to its perspective representation (Fig. 84)?

CHAPTER VIII

SPECIAL PROBLEMS

THE problems in this chapter will be found to be useful in the practical application of perspective.

PROBLEM 42

To determine, without using the C.V. *or* P.D., *the distance from the picture of any point the perspective position of which is given.*

Distance of P.P. = 11′.

Before proceeding with the construction of this problem we

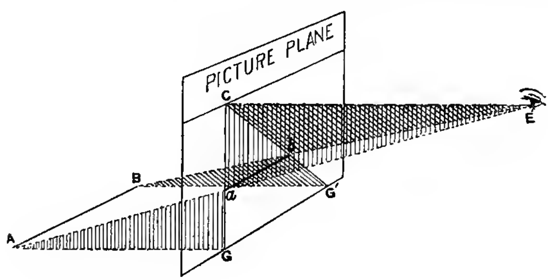


Fig. 92

will consider the principle governing a point in perspective. Fig. 92 is a perspective view in which a line AB is lying on the G.P., and the line *ab* is its perspective representation on the P.P.

A visual ray from the point A intersects the P.P. in the point a, its perspective position. As the line AG is parallel to the line CE (the direction of sight), the two triangles AGa and ECa are similar, consequently—

$$\mathrm{C}a : a\mathrm{G} :: \mathrm{EC} : \mathrm{AG}.$$

Again, a visual ray from B intersects the P.P. in b, and as the line BG′ is also parallel to the line CE, the triangles BG′b and ECb are also similar, therefore—

$$\mathrm{C}b : b\mathrm{G}' :: \mathrm{EC} : \mathrm{BG}'.$$

Let the rectangle abCd (Fig. 93) be the perspective re-

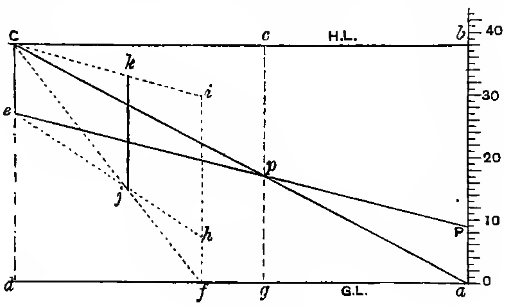


Fig. 93

presentation of a corresponding area of the G.P., and p the perspective position of a given point. The distance of the spectator from the P.P. being 11′. Join aC. Construct a plain scale on the line ab, with the zero point on the G.L., using any unit. In this instance, the given point (p) is placed so as to come on the line aC, in order to simplify the comparison between it and Fig. 92. If it were not on this line, a horizontal line could be drawn from it to meet aC. Draw Ce parallel to ab and make Ce = 11′ from the scale on ab. Join ep, and produce the line to meet ab in P. Then aP (9′) is the required distance of the point.

We will now apply the principle illustrated in Fig. 92 to this problem. Draw a vertical line through p to meet the H.L. and the G.L. in the points c and g. Then $cp : pg :: \mathrm{C}e : \mathrm{P}a$.

Problem 43

To draw a vertical line 4′ long, 7′ from the picture, and 4′ to the spectator's right, without using the P.D. *The spectator being 11′ from the* P.P.

Scale $\frac{1}{4}'' = 1'$. Fig. 93.

Assume C to be the centre of vision. Set off $df = 4'$, and join fC. Draw a vertical line at f, and set off upon it $fh = 7'$, from the scale on ab. Draw a line from h to e, intersecting the line fC in j. Then j is the lower end of the required vertical line, for by the principle illustrated in Fig. 92, $Cj : jf :: Ce : hf$.

Produce fh and make $fi = 4'$, using the $\frac{1}{4}''$ scale. Join iC. Draw a vertical line at j to meet the line iC in k. Then jk is the required vertical line.

Problem 44

To draw from any given point on a vertical line, lines converging towards inaccessible V.P.'s *of two given horizontal lines at right angles to one another.* Fig. 94.

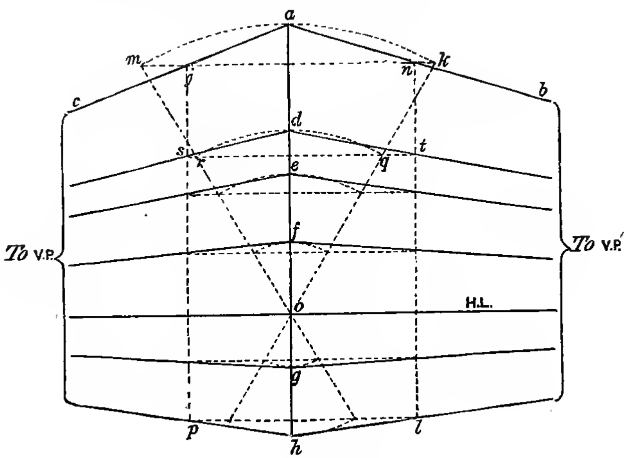


Fig. 94

Let ah be the vertical line, ab and ac the given horizontal lines, and d the given point.

With *o* as centre (the intersection of the given vertical line with the H.L.), and the distance of *a* as radius, draw an arc; at any convenient height draw a horizontal line cutting this arc in the points *mk* and the receding lines in *jn*. Join *m* and *k* to *o*, and draw vertical lines from *j* and *n*. With *o* as centre, and the distance of the given point *d* as radius, draw an arc to meet the lines *mo* and *ko* in the points *r* and *q*. Draw a horizontal line through *rq* to meet the vertical lines from *j* and *n* in the points *s* and *t*. Join *ds* and *dt*, which are the required lines.

The construction for drawing lines from other points, both above and below the H.L., is also shown in the figure.

This problem could also be worked by dividing the lines *jp* and *nl* in the same proportion as *ah* is divided, but the above method will be found more convenient in practice.

Problem 45

1. *The vanishing parallel for one side of a rectangular object, resting on the* G.P., *being given, to determine the direction of its other vanishing parallel.*
2. *From a given point p, to draw a line converging towards the inaccessible* V.P. Fig. 95.

1. Let SV be the given vanishing parallel. With any

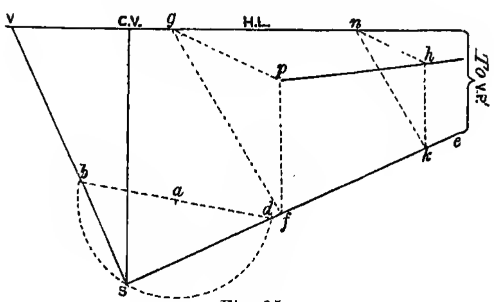


Fig. 95

convenient point *a* as centre, and *a*S as radius, draw an arc cutting SV in *b*. Join *ba*, and produce the line to meet this arc

in *d*. Draw a line from S, through *d*. Then S*d* is at right angles to SV, and is the vanishing parallel required.

2. Assume any convenient point, *f* on the vanishing parallel produced, and *g* on the H.L., and join them; also join *gp* and *fp*. In any convenient position draw *nk* parallel to *gf*. Draw lines from *n* and *k*, respectively parallel to the lines *gp* and *fp*, meeting in *h*. Join *ph*, which is the line required.

Problem 46

To find the M.P. *corresponding to an inaccessible* V.P. *from the direction of its vanishing parallel.* Fig. 96.

Let S*a* be the direction of the vanishing parallel. In any

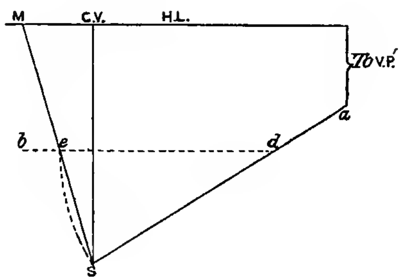


Fig. 96

convenient position, draw a horizontal line *bd*. With *d* as centre, and radius *d*S, draw an arc to meet *bd* in *e*. Join S*e* and produce it to meet H.L. in M, which is the M.P. required [28].

Note.—The three following problems refer to the working points for rectangular objects parallel to the ground plane.

Problem 47

The C.V. *and* V.P.'s *of a perspective drawing being given, determine the station point,* P.D., *and* M.P.'s. Fig. 97

Note.—In the diagrams accompanying this, and the two following problems, the working points that are given are

enclosed in small circles. C represents the centre of vision; V and V′ vanishing points; M and M′ measuring points; S the station point; and D the point of distance.

Bisect the distance between V′ and V in *a*. With *a* as

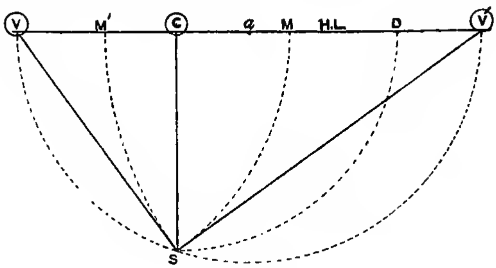


Fig. 97

centre, and radius *a*V, draw a semicircle. Draw a perpendicular to H.L. at C, to meet the semicircle in S. With C as centre, and CS as radius, draw an arc to meet H.L. in D. With V′ and V as centres, and V′S and VS as radii, draw arcs to meet H.L. in M′ and M.

Problem 48

The H.L., *station point, and one* M.P. *being given, determine all the other working points.* Fig. 98.

Draw a line from S, perpendicular to H.L., to find C. Join

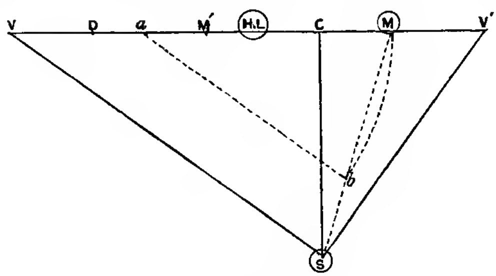


Fig. 98

S to M. Assume any point *a* on H.L. as centre, and with

radius *a*M, draw an arc to meet SM in *b*. Join *ab*. Draw a line from S, parallel to *ab*, to meet H.L. in V. The other V.P. can be found by Prob. 45, the remaining working points by Prob. 47.

Problem 49

The P.D., *and a* V.P. *for lines receding at an angle of* 54° *with the* P.P. *to the right, being given, determine all the other working points.* Fig. 99.

Draw a line through D and V′ for the H.L. Draw a line

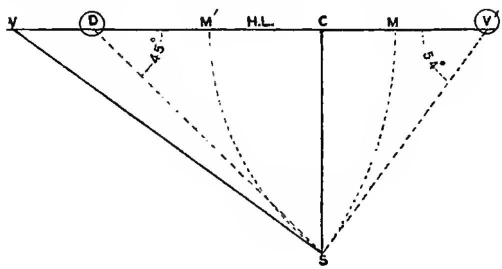


Fig. 99

at V′, making an angle of 54° with this line, also one at an angle of 45° at D. The intersection of these lines will be S, the required station point. A line from S, perpendicular to H.L., will give C. The other V.P. can be found by Prob. 45, the remaining working points by Prob. 47.

Problem 50

To determine, without using the station point, the V.P.'s *and* M.P.'s *for an object whose horizontal lines make with the* P.P. *angles of* 30° *on the right, and* 60° *on the left.*
Distance of P.P. = 8′. *Height of eye* = 4′. *Scale* $\frac{1}{4}'' = 1'$. Fig. 100.

Find the G.L., H.L., C.V., and P.D. (Prob. 4). Draw a perpendicular from C.V., meeting the G.L. in *a*. Set off at *a*, angles of 60° and 30° with the G.L., as in the figure.

In any convenient position draw a horizontal line *bd*. Draw lines from *b* and *d*, at right angles to G.L., and meeting it in the points *e* and *c*. Then *ecdb* is a rectangle. Draw lines from *e* and *c* to C.V., with *c* as centre, and radius *eb*, draw an arc to meet G.L. in the point *f*. Draw a line from *f* to P.D., cutting the receding line from *e* in *b'*. Draw a horizontal line from *b'* to

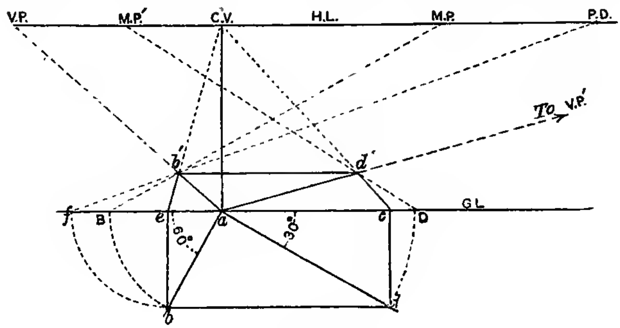


Fig. 100

meet the receding line from *c* in *d'*. Then *ecd'b'* is a perspective representation on the ground plane of *ecdb*. Draw lines from *a* through *b'* and *d'* to meet H.L., and we get V.P. and V.P.', the vanishing points required.

With *a* as centre, and radius *ab*, draw an arc to meet G.L. in B. Draw a line from B through *b'* to meet H.L., and we get M.P. Find M.P.' in a similar manner.

Problem 51

Draw, without using either the station point or M.P., *a rectangular solid* 3' × 2' × 1', *resting with one of its faces on the* G.P., *its longest edges receding to the right at an angle of* 30°, *and its shortest edges vertical. Its nearest angle is to be* 4' *from the picture, and* 3' *to spectator's right.*

Distance from P.P. = 8'. *Height of eye* = 3'. *Scale* $\frac{1}{2}'' = 1'$. Fig. 101.

Find the V.P.'s as described in the preceding Prob. With

a as centre, and a radius of one foot, draw an arc cutting the plan of the given angles in *b* and *c*, and the G.L. in *d*.

Draw lines from *c* and *b*, at right angles to G.L., and meeting it in the points *f* and *g*. Draw lines from *d*, *f*, and *g* to C.V.

Then the receding lines drawn from *d* and *a* form a perspective scale for feet parallel to the picture, and those from *fa*

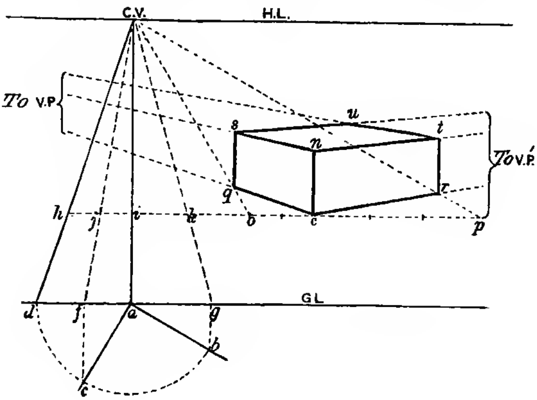


Fig. 101

and *ag* are perspective scales for the picture size of the feet receding respectively to the left and right.

Find the position of the nearest angle of the object *e* (Prob. 5). Draw a horizontal line through *e*.

Draw the perpendicular *en* equal to *hi*, and draw lines from *e* and *n* to each V.P.

Set off *ep* equal to three times the perspective foot *ik*, and *eo* to twice the perspective foot *ji*. Join *p* and *o* to C.V., cutting the receding lines from *e* in the points *r* and *q*. Draw perpendicular lines at these points, and complete the solid as shown.

We could also measure a given length on a receding line by means of the station point, *e.g.* to measure a length of 3′ on a line receding from the point *a* to its V.P. (Fig. 102).

Draw *ad* parallel to SV — the vanishing parallel for the

receding line from a — and three times the length of ef (the perspective foot at the distance of a). Draw a line from d to S, cutting the receding line from a in b. Then ab is the length required. The reason for this will be apparent in the following Prob. If we measure the line ab in the usual manner, by its

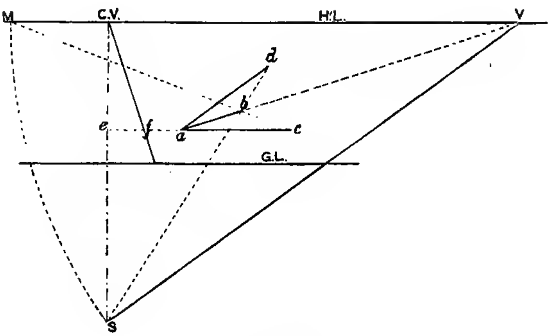


Fig. 102

M.P., we shall find that its length will coincide with that already obtained, as shown.

It is possible to determine the apparent length of a line in perspective by calculation only. We have shown in Fig. 92 the principle governing a point in perspective by similar triangles. On the same principle we could calculate the position of any two points on the P.P., by joining which we could obtain a line in perspective.

Let C (Fig. 103) be the centre of vision, S the station point 6′ from the P.P., and D the point of distance, the scale being $\frac{1}{4}'' = 1'$.

From any point a on the G.L. draw a line to C. Measure off a length of 2′ on this line, by setting off $ac = 2'$, and drawing a line from c to D cutting the receding line from a in b.

Then $ac : \text{CD} \, (= \text{SC}) :: ab : b\text{C}$, and $cb : b\text{D} :: ab : b\text{C}$.

So if we assume $a\text{C}$ to $= ca + \text{CD}$, we could divide it in the same proportion, and obtain the position of b without the construction: *e.g.* $\text{CD} = 6'$, and $ca = 2'$; so $\text{CD} + ca = 8'$, of which

sum $ca = \frac{1}{4}$; consequently, if we make $ab = \frac{1}{4}$ of aC, we determine the position of a, the triangles abc and CbD being similar.

Let us draw a line, 2 feet long, receding from any point e on the G.L., at right angles to the P.P. Join eC, and assume, as before, this line to equal the length of the line we wish to

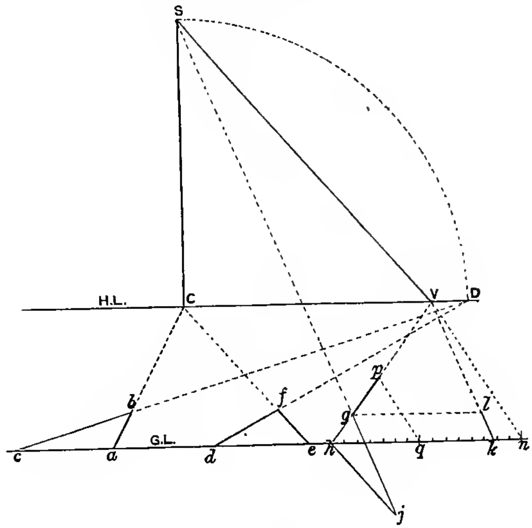


Fig. 103

draw + the distance of the P.P. ($2' + 6' = 8'$). Then, by making $ef = \frac{1}{4}$ of eC, we obtain the required line ef.

Draw a line from D, through f, to meet the G.L. in d.

Then de : CD :: ef : fC, and df : fD :: ef : fC, ∴ the triangle def is similar to the triangle DCf.

The same principle could be applied to any V.P., as will be seen from the following:—

With S as centre, and a radius $= 8'$, mark off on the H.L. a point V. Join SV, which is the vanishing parallel for all lines drawn to V. From any point h on the G.L. draw a line hj, $2'$ long, parallel to SV. Join h to V, and j to S, intersecting hV in g. Then hj : SV :: hg : gV, and jg : gS :: hg : gV, ∴ the

triangle hjg is similar to the triangle VSg. We know that $hj = \frac{1}{5}$ of $SV + hj$, so we could have obtained the position of g by making $hg = \frac{1}{5}$ of hV.

We wish to draw a line 2′ long from any point k on the G.L., and receding towards the point V. Join kV. As any line drawn from the G.L. to any V.P. on the H.L. may be assumed to equal the length of the vanishing parallel + the distance of a given point from the P.P., on a line drawn to the V.P., by dividing kV in this proportion we obtain the point l. Then kl is the required line.

Problem 52

Draw in perspective, by calculation, a line 5′ long, lying on the G.P., and receding towards a given V.P., its nearer end to be 2′ from the P.P. measured in the direction of the given line.

To simplify the construction, let us assume g (Fig. 103) to be the nearer end of the required line, and V the given V.P.

As the line is to be 5′ long, its farther end will be 7′ from the G.L., measured in the direction of the given line. The length of the vanishing parallel (8′) + the distance of the farther end of the line from the G.L. (7′) = 15′. So, if we measure $\frac{7}{15}$ of the line hV from h, we obtain the farther end of the required line. This is best done in the following manner :—

Set off from h, on the G.L., fifteen equal divisions of any convenient length, and join the last point (n) to V. From the end of the seventh division from h (q) draw a line parallel to nV, to meet the line hV in p. Then gp is the required line.

The triangles hqp and hnV are similar. Therefore $hp : pV :: hq : qn$.

Problem 53

The top of a square tower being given in perspective, with the position of the H.L., *determine the* V.P.'s, C.V., *station point, and the angles the sides of the tower form with the picture.* Fig. 104.

Let *cab* be the top of the tower; produce *ab* and *ac* to meet the

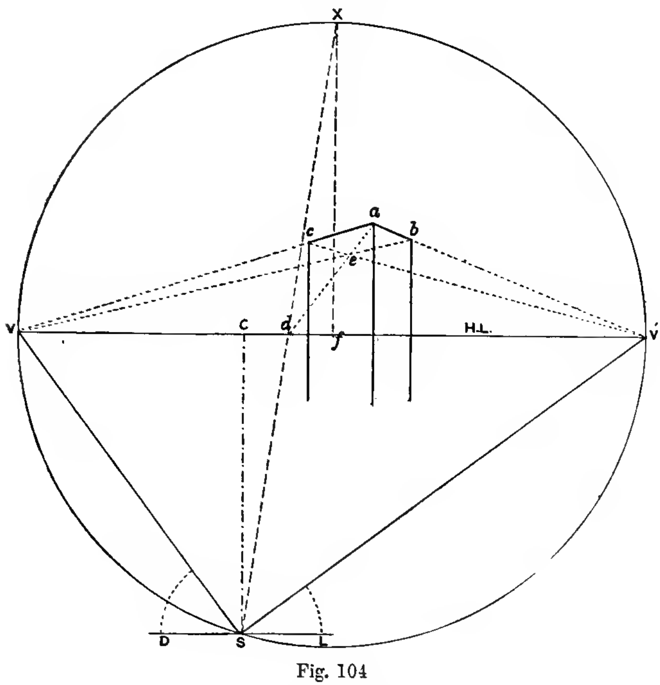


Fig. 104

H.L., to determine the vanishing points V and V′ Draw a line from *b* to V, and from *c* to V′, intersecting in *e*. Join *ae*, and produce the line to H.L. This will determine the diagonal vanishing point *d* [43].

Bisect the line VV′ in *f*. With *f* as centre, and *f*V as radius, draw a circle. Draw a line at *f*, at right angles to VV′

to meet the circumference of the circle in X. Draw a line from X through *d* to meet the circumference of the circle in S.

S is the station point, and a line drawn from S at right angles to VV′ will give C, the centre of vision.

The angles VSX and V′SX are equal because they stand upon equal arcs.

Draw a horizontal line (DL) through S. Then VSD and V′SL are the angles the sides of the tower form respectively with the picture.

Problem 54

1. *Let* ab *be the perspective diagonal of a square, lying on the* G.P., C.V. *the centre of vision, and* S *the station point. Draw the complete square.*
2. *Determine the angles the sides of the square form with the picture.*
3. *Assume the sides of the square to be* 6′ *long and the scale* $\frac{1}{8}'' = 1'$. *Determine the position of the* G.L.
4. *Determine the true position of the nearest angle of the square, i.e. its distance from the picture and its distance to spectator's right or left.* Fig. 105.

N.B.—This problem is the type of an important class.

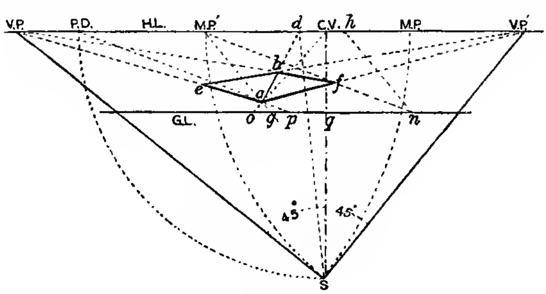


Fig. 105

1. Produce *ab* to meet H.L. in *d*. Then *d* is the diagonal vanishing point [43]. Join *d* to S. Draw lines from S at an

angle of 45° with Sd, and produce them to meet H.L. in V.P. and V.P.′ Draw lines from a, and through b to each V.P., forming the figure $afbe$, which will be the perspective square required.

2. The lines drawn from S to each V.P. will determine the angles the sides of the square form with the picture, as explained in the preceding Prob.

3. Find the M.P.'s (Prob. 47). Draw a line from M.P.′ through a to meet G.L. in g. Set off 6′ on H.L. from M.P.′ to find h. Draw a line from h, parallel to the line from M.P.′, to a, meeting a line drawn from M.P.′ through f in n. Draw a horizontal line through n for the G.L.

4. Draw a line from S, perpendicular to H.L., to find C.V. With C.V. as centre, and the distance of S as radius, draw an arc to meet H.L. in P.D. Draw lines from P.D. and C.V. through a to meet G.L. in the points p and o. Then op is the distance of the nearest angle of the square from the picture, and oq is its distance to spectator's left.

The perspective length of a line, parallel to the picture, is to its actual length as the distance of the picture is to the real distance of the line from the spectator : *e.g.* Let AB (Fig. 106) represent a line, P the position of the picture, and E the position of the eye.

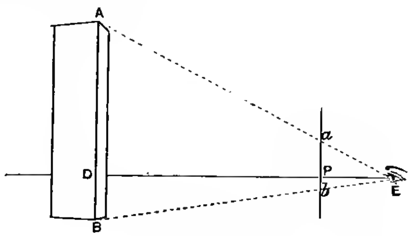


Fig. 106

Then, by similar triangles, ab : AB :: EP : ED.

Problem 55

The position in which the model (Fig. 107) *meets the* G.P. *is unknown, but the height of* ab *is known to be* 3′. *Determine the distance of* ab *from the picture. Scale* $\frac{1}{8}'' = 1'$. Fig. 107.

Determine the position of the centre of vision (C) (Prob. 53), and the point of distance (D) (Prob. 47).

Draw a perpendicular through C, and assume it to be the edge of the picture plane. Set up CE equal to *ab*, also CF equal to 3′ by the scale. Draw a line from D, through E, to meet a horizontal line from F in G. Draw the vertical line GK. Then CK is the distance required.

For CE : KG : : DC : DK. Therefore, by the proposition first proved, CK must be the distance of *ab* from the picture.

The distance of an object from the eye being fixed, the

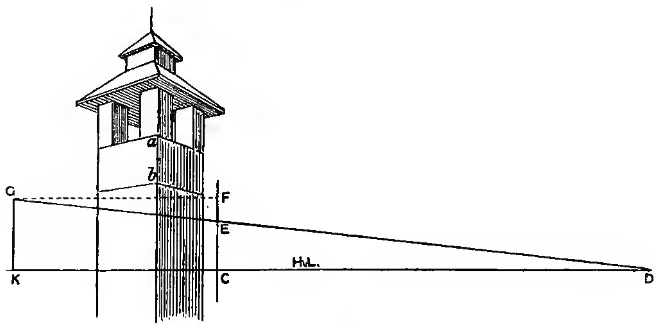


Fig. 107

scale of the drawing will depend upon the position of the P.P. So to determine from the perspective the distance of the real object from the picture we must know the scale to which the drawing is made [50].

Problem 56

The supplementary sketch in Fig. 108 *is taken from the view shown in* Fig. 40. *The small turret* A *is known to be* 4′ *wide. Determine its distance from the spectator, on a line at right angles to the* P.P., *as well as its distance from the* P.P. Fig. 108.

Assume C.V. and P.D. to be respectively the centre of vision and point of distance of the drawing.

Note.—The P.D. would really come beyond the limits of this diagram.

Draw a vertical line from C.V., and in any convenient position on this line draw *ab* at right angles to it, and equal in length to the width of the turret A in sketch.

First assume the scale of the drawing to be $\frac{1}{20}'' = 1'$. Produce *ba* and make *bc* equal to 4′ of this scale. Draw a perpendicular from *c* to meet a line from C.V. through *a* in *e*. Draw a horizontal line through *e*.

Draw a line from P.D. through *b* to meet this horizontal line in *f*. Then *hf* is the distance of the turret from the spectator to this scale (52′), on a line at right angles to the P.P., and *gf* its distance from the picture (19′).

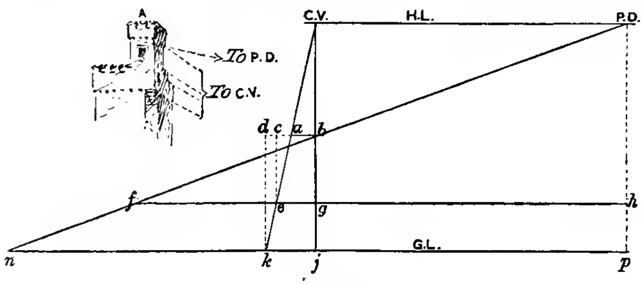


Fig. 108

We will now assume the scale of the drawing to be $\frac{1}{16}'' = 1'$. Make *bd* equal to 4′ of this scale, and draw a perpendicular from *d* to meet *ae* produced in *k*. Draw a horizontal line through *k*. Produce the line from P.D. through *b* to meet this line in *n*.

Then *pn* is its distance from the spectator (52′) and *jn* its distance from the picture (26′).

It will be seen that although we vary the scale the distance of the object from the spectator remains the same (52′), so that the position of the P.P. determines the scale of the drawing.

CHAPTER IX

PERSPECTIVE FOR SCIENCE AND ART STUDENTS

ADVANCED STAGE

Introduction

Note.—The articles in this chapter are numbered in continuation of those in Chapter II.

The numerals in brackets refer to the numbered articles in Chapter II., and continued in this chapter.

We have already briefly stated what vanishing lines [33] and accidental vanishing points are [39]; we will now go more fully into the subject.

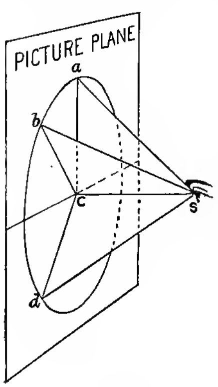


Fig. 109

51. There may be an infinite number of vanishing points for lines forming the same angle with the picture plane, with the exception of those at an angle of 90°.

All lines at 90° with the picture plane must be parallel to one another, but this is not the case with any other angle, as will be seen from the following illustration.

Let S (Fig. 109) be the station point, C the centre of vision, and aSC an angle of 45°.

52. If we assume SC (the direction of sight) to remain fixed, and Sa to move so as always to make the same angle with SC, the line Sa would generate a right cone, of which SC would be the axis; consequently, the line Sa is called a *genera-*

trix, and since, evidently, S*a* will make a constant angle of 45° with the picture plane at every point of its rotation round the axis SC, so every point, as *b* and *d* in the circumference of the circle forming the base of the cone, is a vanishing point for lines

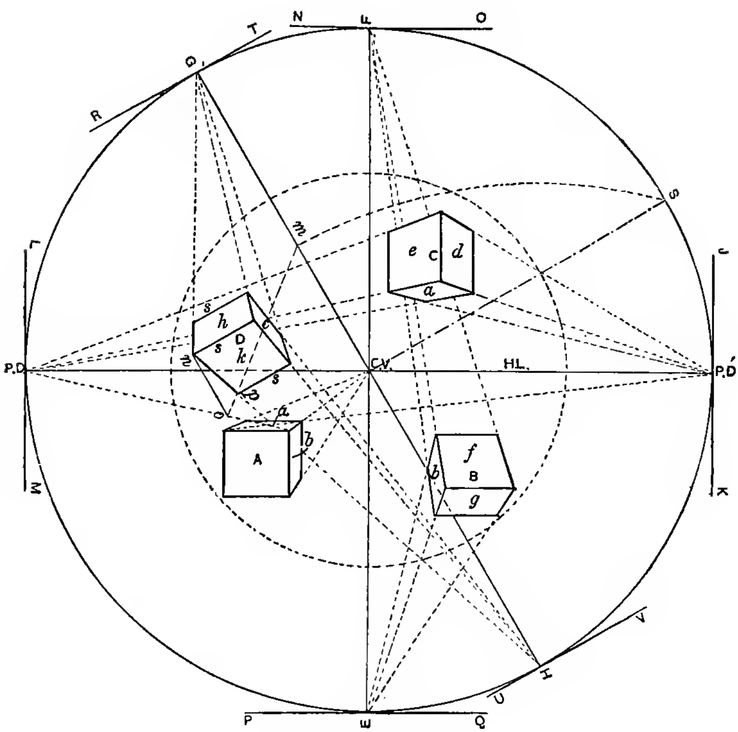


Fig. 110

at 45° with the picture plane [27]. The same principle applies to every vanishing point, the centre of vision excepted.

53. It will be seen from this Fig. (109) that a line at any angle with the picture or ground planes may be contained by a plane at right angles to the picture plane, because it is the generatrix of a right cone.

Note.—The advanced stage of perspective will be more easily understood if we assume every vanishing point to be

determined by the generatrix of a cone, the centre of vision, of course, excepted.

Fig. 110 shows a cube drawn in perspective in four different positions; the outer circle of the figure is the base of a cone, the generatrix of which is inclined at an angle of 45° to the picture plane. The centre of the circle (C.V.) being a point opposite the spectator's eye, a horizontal line through it must be the horizon line [18]; the points P.D. and P.D.′ are the points of distance [23], and, being on the horizon line, each must be a vanishing point for a certain series of horizontal lines [20]; and as they are also on the circumference of the circle, they must be the vanishing points for all horizontal lines inclined at an angle of 45° to the picture plane. The diagonals of the top of the cube A, as well as the receding lines of the cube C, converge towards these points.

This figure should be well studied, as it illustrates the relation that exists between vanishing points and vanishing lines.

The cube A has two faces parallel to the P.P., and therefore two parallel to the G.P.

The cube B has four edges parallel to the G.P., but no face parallel either to the G.P. or to the P.P.

The cube C has four faces vertical, but no face parallel to the P.P.

The cube D has no face parallel either to the P.P. or to the G.P.

All four of the cubes are so drawn as to have some of their edges parallel to the picture plane: *e.g.* A has all its vertical as well as four of its horizontal edges; B has its four horizontal edges only; C has its four vertical edges only; and D has the edges marked *s*; consequently, all these edges in each cube are drawn, of course, geometrically parallel to each other [21].

The vanishing line for the faces marked *a* is the horizon line, as they are contained by horizontal planes [35], and all receding lines forming parts of these faces must have their vanishing points on this line.

All faces marked *b*, being in vertical planes perpendicular to the picture plane, must have a vanishing line at right angles to the horizon line, and passing through the C.V., as EF [34]; and all receding lines forming parts of these faces must have their vanishing points on this line [39]. The same principle applies to the following faces:—GH drawn through C.V., at right-angles to the nearest edge of the cube D, is the vanishing line for the face *c*; for this face, being perpendicular to the picture plane, its vanishing line must pass through the C.V. [34]. JK is the vanishing line for face *d*, and LM for face *e*. Since both these faces are at right angles to the face *a* of the cube C, their vanishing lines must also be at right angles to the vanishing line of the face *a*, *i.e.* the horizon line.

54. The intersections of these vanishing lines must be the vanishing points of the edges formed by the intersection of the corresponding edges of the cube. The same principle applies to the faces of the other cubes, *e.g.* NO is the vanishing line for the face *f*, and PQ for the face *g*, both of which lines are at right angles to the vanishing line EF, and the edges formed by the intersection of pairs of faces of the cube will converge towards the points formed by the intersection of the corresponding vanishing lines. RT is the vanishing line for the face *h*, and UV for the face *k*, both of which lines are at right angles to the vanishing line GH.

The dotted circle is the field of vision [44].

Fig. 110 also illustrates all the planes that occur in perspective with their vanishing lines.

55. The front of the cube A is parallel to the picture plane, consequently it has no vanishing line.

The faces of the cubes marked *a* are contained by *horizontal planes*, for which the horizon line is the vanishing line [35].

56. The faces marked *b* are contained by *vertical planes at right angles to the picture plane*, the vanishing line for which is a vertical line through the centre of vision [34].

57. The face *c* of the cube D is contained by an *inclined*

plane at right angles to the picture plane, consequently its vanishing line must pass through the centre of vision [34].

Note.—Planes inclined to the ground plane, and at right angles to the picture plane, are called *inclined planes.*

58. The faces *d* and *e* of the cube C are *vertical planes inclined to the picture plane.*

59. *Note.*—The vanishing line for a vertical plane is always vertical, and must pass through the vanishing point for any horizontal line in that plane.

60. A plane parallel to the horizon line, such as the plane containing the face *f* of the cube B, is called an *ascending*

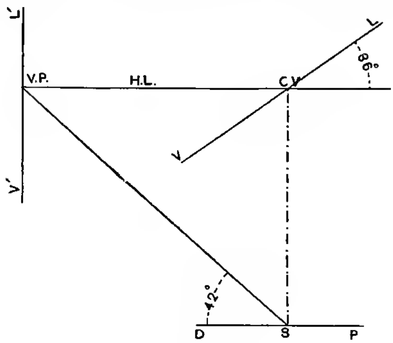


Fig. 111

plane, and a similar one containing the face *g,* a *descending plane.* Horizontal lines in these planes are always parallel to the picture plane [38].

61. *Note.*—The vanishing lines of all ascending and descending planes are horizontal.

62. Planes that are parallel to neither a vertical line nor to a horizontal line are called *oblique planes.* The faces *h* and *k* of the cube D are contained by *oblique planes.*

63. Lines that are parallel neither to a vertical line nor a horizontal line are called *oblique lines, e.g.* those converging towards the points E, F, G, H.

We have shown how the vanishing lines for *ascending*

and *descending* planes are determined from given data in Article 38.

64. To find the vanishing line for an *inclined plane* forming an angle with the ground plane (*e.g.* an angle of 36° on the right), we simply draw a line (V.L.) through the C.V. at this angle with the horizon line, as shown in Fig. 111, and this is the vanishing line required.

65. To find the vanishing line for a *vertical plane inclined to the picture plane*, at any angle (*e.g.* an angle of 42° on the left), set off at the station point this angle with the directing line (Fig. 111), and let the bounding line of the angle meet the horizon line in V.P. Draw a vertical line V.′L.′ through V.P., as shown. This is the vanishing line required.

66. We have shown [53] that any line may be contained by

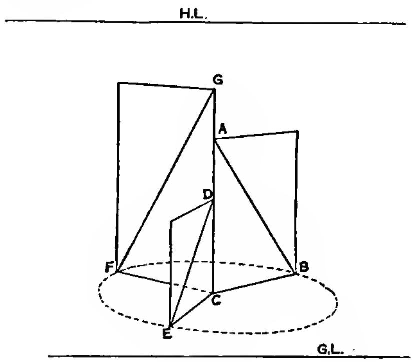


Fig. 112

a plane at right angles to the picture plane, the vanishing line of which must pass through the centre of vision [34]. In a similar manner, a line of the same character may be contained by a vertical plane, the vanishing line for which is vertical [59].

The three lines AB, DE, and FG (Fig. 112), may be assumed to be the generatrices of three cones having a common base in the dotted circle. It is evident that these three generatrices must form different angles with the ground plane, because they are each at the same distance from their common axis on the

ground plane, while the vertex of each cone is at a different height.

These three generatrices may be contained by vertical planes forming different angles with the picture plane, as shown.

We will take the plane containing the diagonal AB as an illustration. The line CB is the intersection of this plane with the ground plane, and if produced to meet the horizon line, would determine the vanishing point for all lines parallel to it [20]. Now it has been clearly shown [54] that the vanishing point for lines common to two planes must also be the point of intersection of the vanishing lines for those planes, therefore the vanishing point for the line CB must be the point where the vanishing line for the plane containing the line AB intersects the vanishing line for the ground plane, *i.e.* the horizon line. As all receding vertical planes must have vertical vanishing lines [59], a vertical line drawn through the vanishing point for the line CB would be the vanishing line of the plane containing the line AB. The vanishing lines for the two other planes would be determined in a similar manner.

67. Accidental Vanishing Points.—Receding parallel lines inclined to the ground plane will converge towards an *accidental vanishing point* [39]; these points are found in a similar manner to ordinary vanishing points. The following rule is applicable to every kind of vanishing point:—

68. *To determine the position of a vanishing point for any given line, draw a line parallel to it from the eye to meet the picture plane; this line, drawn from the eye, is called the vanishing parallel of the given line, and its intersection with the picture plane is the vanishing point required* [13].

69. Directing Plane.—When receding lines are parallel to the ground plane, we set off the angle of their vanishing parallel [13], at the station point, with the directing line [31]; but when they are inclined to the ground plane, it is necessary to assume a vertical plane passing through the station point parallel to the picture plane. Such a plane is called the

directing plane, and is always at right angles to the direction of sight.

Fig. 113 is a perspective sketch showing the position of the directing plane. SV is the generatrix of a cone [52], *i.e.* the vanishing parallel [13] for the required line.

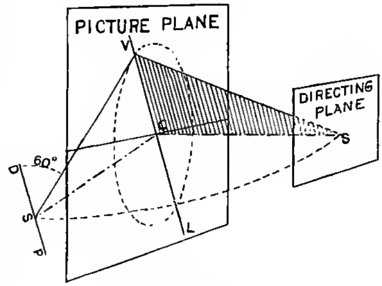


Fig. 113

Let us assume that the line SV (Fig. 113) is contained by a plane at right angles to the picture plane, and inclined to the ground plane at an angle of 50° on the left, and that the line SV is inclined to the picture plane at an angle of 60°.

We will now determine the accidental vanishing point for this line, as well as the vanishing line of the plane containing it.

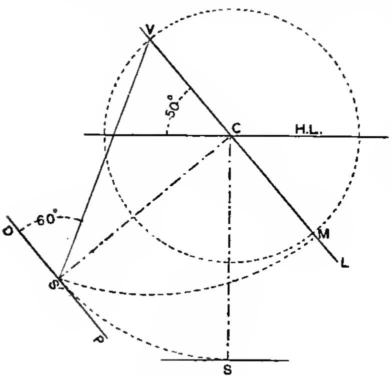


Fig. 114

Let H.L. (Fig. 114) be the horizon line [18], S the station point [11], and C the centre of vision [14]. Set off at C an angle of 50° on the left from H.L., and draw a line VL through it [34]. If we refer to Fig. 113, we shall see that the line CS must be at right angles to the line VL; so draw a line from C (Fig. 114) at right angles to the line VL. With C as centre, and radius CS, draw an arc to meet this line in S′. Now CS′ is the direction of sight [15] "constructed" on to the picture plane, and as the directing plane is at right angles to this line [69], draw a line DP through S′, and at right angles to S′C, to represent the edge of this plane. Draw a line making the angle required (60°) at S′,

with DP, and produce it to meet VL in V. Then the line S'V is the generatrix of the cone "constructed" on the picture plane. We have, in fact, "rotated" the triangle SCV (Fig. 113) on the line CV to coincide with the picture plane, as shown.

70. ACCIDENTAL MEASURING POINTS.—Accidental vanishing points, like ordinary vanishing points, have corresponding measuring points [28], called *accidental measuring points*, which are found by the following general rule :—

71. *To find the measuring point corresponding to any vanishing point, take the distance of the station point from the vanishing point, and set it off from the* V.P. *on the vanishing line of a plane containing the line we wish to measure.*

So with V as centre (Fig. 114), and VS' as radius, draw an arc to meet VL in M, the accidental measuring point required.

72. To show how an accidental measuring point is used for measuring a receding line, we will refer to Fig. 110. Here *m* is the measuring point corresponding to the vanishing point H, and is found as just described. We wish, for example, to measure a length on the receding edge of the cube D, equal in length to its nearest edge. Set off this distance, as *no*, on a line parallel to the vanishing line of the plane containing the line to be measured (GH), and draw a line from *o* to *m*, cutting the receding line from *n* in *p*. Then *np* is equal to *no*.

The principle of these accidental measuring points is precisely the same as that governing the use of ordinary measuring points, and fully described in Article 29, *i.e.* they are really the vanishing points for the bases of isosceles triangles, *e.g.* *np* (Fig. 110) is equal to *no*, therefore *nop* is an isosceles triangle of which *op* is the base.

VANISHING LINES [33]

The following may be taken as a general rule for all vanishing lines :—

73. *To determine the vanishing line for any receding plane, assume a plane parallel to it to pass from the station point to*

meet the picture plane ; its intersection with the picture plane is the vanishing line required.

Let ABLV (Fig. 115) represent a plane parallel to a given plane—called an *original plane*—passing through the station point, and meeting the picture plane in the line VL. This line VL is the vanishing line for all planes parallel to the plane ABLV.

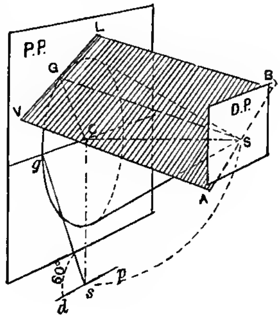


Fig. 115

74. Assume this plane to be in contact with a cone, the vertex of which is the station point (S). In this position it is called a *tangent plane* to the cone, and they touch each other in a straight line (SG), called the *line of contact.* A line drawn from C to meet VL at right angles will always meet it in the same point as the line of contact.

75. The line of contact may be taken as the generatrix of the cone, and is a similar line to SV, illustrated in Figs. 113 and 114, and by its means we can determine the angle the oblique plane forms with the picture plane, *e.g.* let S (Fig. 116) be the station point, H.L. the horizon line, and V.L. the vanishing line. Draw a line from S at right angles to H.L. to find the centre of vision (C). Draw a line from C at right angles to V.L. meeting it in G. With C as centre, and CG as radius, draw an arc to meet H.L. in *g*. Join *g*S, which is a generatrix of the cone. Draw DP through S parallel to H.L. for the directing plane [69]. Then *g*SD is the angle the oblique plane forms with the picture plane. This construction is also shown in the perspective sketch (Fig. 115).

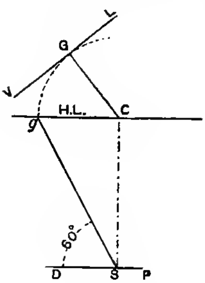


Fig. 116

76. The plane *abcd* (Fig. 117) is a vertical plane inclined to the picture plane at an angle of 60° to the right; consequently,

V′ is the vanishing point for its horizontal edges [20]; and a vertical line drawn through V′ is the vanishing line for all planes parallel to it [59].

77. Draw the diagonal *db*, and produce it to meet this vanishing line in A, which is the vanishing point for all lines parallel to *db* [39].

78. We wish to determine the angle the line *db* forms with

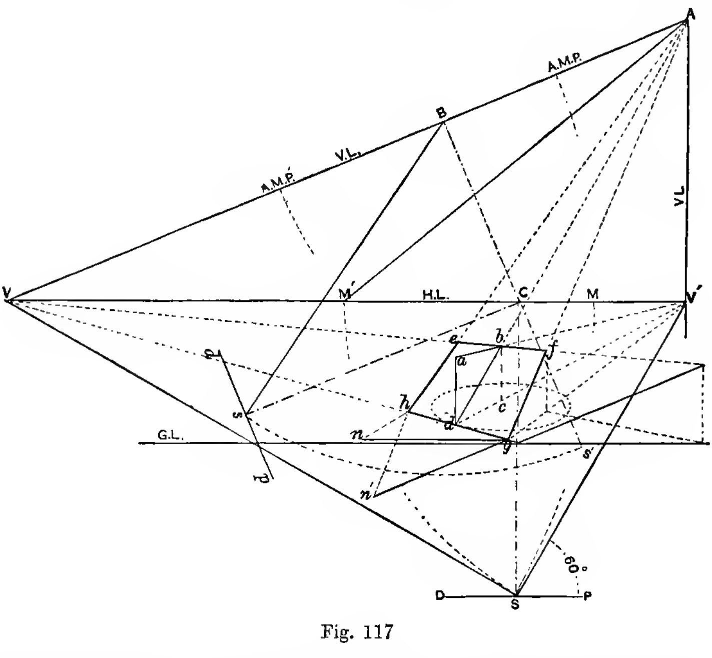


Fig. 117

the ground plane. First determine the measuring point corresponding to V′, as M′ [28]. Join A to M′. Then AM′V′ is the angle required; because, if we assume the right-angled triangle AM′V′ to be contained by a vertical plane on the line SV′, and rotate it on the edge V′A, from coincidence with the right-angled triangle ASV′, till it coincides with the triangle AM′V′ on the picture plane, the line M′A represents for the line *db* the vanishing parallel "constructed" on the picture plane, in a similar manner to the line S′V in Fig. 114.

79. Let *efgh* be an oblique plane at right angles to the plane *abcd*, and let *db* be the line in which they intersect. If these planes are at right angles to each other, their lines of intersection with the ground plane must also be at right angles to each other; so the line *gh* must be inclined to the picture plane at an angle of 30°, and, being horizontal, must have its vanishing point at V.

80. We have now the vanishing points for two lines contained by the plane *efgh*, viz. A and V, by joining which we obtain the vanishing line for the plane. The point A is called the *vanishing point of the inclination* of the plane *efgh*.

81. To determine the angle this plane forms with the picture plane, draw a line from C at right angles to its vanishing line, meeting it in the point B. Draw C*s* at right angles to CB and equal in length to CS. Join *s*B. Draw *dp* at right angles to C*s*. Then B*sd* is the angle the plane forms with the picture plane.

The line *s*B can either be taken as the generatrix of a cone (tangential to the oblique plane) "constructed" on the picture plane, as shown in Fig. 114, or it may be taken as the line of intersection between the oblique plane and a plane at right angles to it, passing through the centre of vision, also "constructed" on the picture plane. The student should compare the construction for finding this line with that for the line S′V (Fig. 114); and also that for the line S*g* (Fig. 116).

It will be seen that the points V and A are the vanishing points for lines at right angles to each other, viz. the lines *gh* and *gf*. We will now determine the accidental measuring points corresponding to these vanishing points.

82. With B as centre, and radius equal to B*s*, draw an arc to meet BC produced in *s*′. Then B*s*′ is the distance of the station point from the vanishing line VA, on a line at right angles to it. With V and A as centres, and the distance of *s*′ from each point as a radius, draw arcs to meet the vanishing line in the points A.M.P. and A.M.P.′ These are the accidental measuring points required.

83. Let us now determine the length *gh* on a line parallel to the picture plane at the distance of *g*. Draw a line from its measuring point M, through *h*, to meet a horizontal line from *g* in *n*. Then *gn* is the length required. Suppose we wish to measure the length *gh* by means of the accidental measuring point (A.M.P.), on the line *gh*; draw a line $gn' = gn$ parallel to the vanishing line VA, and draw a line from *n'* to A.M.P. This will be found to meet *gh* in *h*.

84. Traces of Planes.—The line in which a plane produced

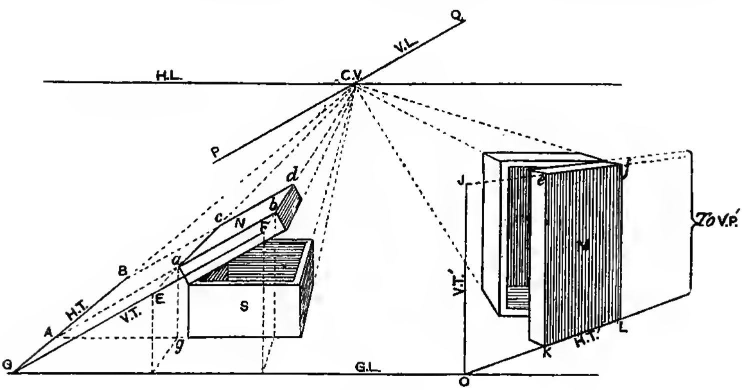


Fig. 118

meets the ground plane is called its *horizontal trace*, and the line in which a plane produced meets the picture plane is called its *vertical trace*. Vertical traces are also called *picture lines* [40].

We shall find the small box we have used in previous illustrations very useful to illustrate these traces. Place it with its lid slightly raised, and one of its ends parallel to the picture plane, as S (Fig. 118). The face N is contained by an inclined plane [57], and if produced would meet the ground plane in the line AB (its horizontal trace), and the picture plane in the line EF (its vertical trace).

Now place the box with one of its ends on the ground plane, so that the lid is inclined to the picture plane as M (Fig. 118). In this position M is a vertical plane inclined to the picture

plane, KL is its horizontal trace, and if the plane containing it were produced to meet the picture plane, JO would be its vertical trace.

Next place it with its back parallel to the picture plane as S (Fig. 119). The face N in this position is contained by an ascending plane, of which AB is the horizontal trace, and EF the vertical trace.

Now reverse its position, so that its front is parallel to the picture plane as T (Fig. 119). In this position the face M is a

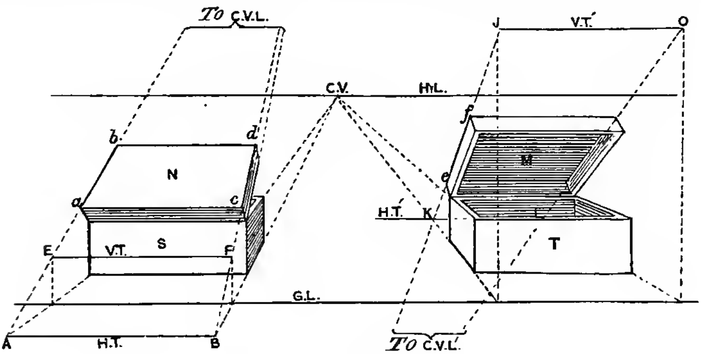


Fig. 119

descending plane, and KL is the horizontal trace, and JO is the vertical trace of a plane containing it.

Now place it resting on the ground plane with its sides inclined to picture plane, as shown in Fig. 120. In this position the face N is an oblique plane, AB is the horizontal trace, and GF the vertical trace of a plane containing it.

85. There are a few points with reference to these traces that are deserving of attention: viz. Vanishing lines of planes are always parallel to their vertical traces, *e.g.* in each of the Figs. 118 and 120, PQ is the vanishing line for the plane N, and is parallel to GF, its vertical trace. The vanishing lines for the other planes are beyond the limits of the picture, but they would be parallel to their vertical traces.

In ascending and descending planes (Fig. 119) the hori-

zontal and vertical traces are parallel to each other; in every other kind of receding plane the horizontal and vertical traces meet at the ground line (see Figs. 118 and 120).

86. Any plane drawn at right angles to the horizontal trace of a plane will obviously give, by its intersection with that plane and the ground plane, the inclination of that plane to the ground plane. Similarly any plane drawn at right angles to the vertical trace of a plane will give, by its intersection with that plane

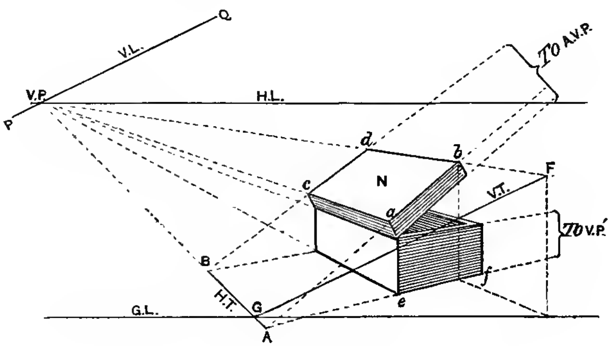


Fig. 120

and the picture plane, the inclination of that plane to the picture plane.

87. Centre of Vanishing Line.—The vanishing point for lines parallel to receding planes, and at right angles to the vertical traces of the planes, is called the *centre of the vanishing line;* because, as the vanishing line of a plane is parallel to its vertical trace [85], the line must also be at right angles to the vanishing line, *e.g.* the centre of vision is the centre of the vanishing lines for all planes at right angles to the picture plane (see Fig. 110). Also in Fig. 117, V′ is called the centre of the vanishing line V′A, and B is called the centre of the vanishing line VA.

88. The centre of a vanishing line is determined by drawing a line from the centre of vision to meet it at right angles. The vanishing line for the oblique plane (Fig. 117) has been

determined by joining the vanishing points (V and A) of two lines contained by the plane; and we have had to assume a vertical plane (*abcd*) containing one of these lines to find one of these vanishing points (A); the other line (*hg*) is a horizontal

Fig. 121

line in the oblique plane—its horizontal trace. This method simplifies the construction; but it is possible to find the vanishing line for an oblique plane, if the angles it forms with the picture and ground planes are given, without having to find vanishing points for lines contained by the oblique plane. Also in the

vanishing point V (Fig. 114), determined by the vanishing parallel S′V, we have assumed the vanishing parallel to be contained by a plane at right angles to the picture plane, as shown in the perspective sketch (Fig. 113). This vanishing point could also be found, if the angles the line forms with the picture and ground planes were given; but it is advisable for the student, in the first place, to work out the problems given for the different planes as illustrated in this chapter; consequently, we will defer the explanation of this latter method.

Note.—The numbered articles are continued in Chapter XI.

Students desirous of putting their knowledge of perspective to a practical test, should endeavour to work out the following exercises in connection with Fig. 121 :—

EXERCISES

Practice

Note.—To work out these exercises, the necessary lines should be first traced from the illustration, and then transferred to a sheet of paper.

1. Determine the angle the front of the building forms with the picture plane (Prob. 53).
2. Determine the position of the centre of vision, and the distance of the picture plane (Prob. 53).
3. Determine the accidental vanishing point for the line AB [76 and 77].
4. Determine the angle the line AB forms with the ground plane [78].
5. Determine the vanishing line for a plane containing the lines CD and EF [76 and 77].
6. The height of GH is known to be 30 feet, and the scale of the drawing $40' = 1''$, determine the distance of the line GH from the picture plane (Prob. 55).
7. Determine the vanishing line for an oblique plane containing the line AB [80]. Find the angle this plane forms with the picture plane [81]; also its angle of inclination to the ground plane [78].

Theory

1. In what way does a line making an angle of 90° with the

P.P. differ with regard to its V.P. from a line making any other angle [51]?

2. What is the *generatrix* of a cone, and why is it so called [52]?
3. What is an *ascending plane* [38]?
4. What is a *directing plane* [69]?
5. What is an *oblique line* [63]?
6. What is a *descending plane* [38]?
7. What is an *inclined plane* [57]?
8. What is an *oblique plane* [62]?
9. What are *accidental vanishing points* [39], and how are they found [67]?
10. What are *accidental measuring points* [70]; and how are they found [71]?
11. What are *vanishing lines* [33]? State a general rule for finding vanishing lines [73].
12. What are *traces* of planes [84]?
13. What are *vertical traces* of planes; and what other name is applicable to them [84]?
14. What is a *horizontal trace* of a plane [84]?
15. What is the *centre of vanishing line* [87]?
16. What is the *vanishing point of the inclination of a plane* [80]?

CHAPTER X

SOLIDS IN COMPLEX POSITIONS

THE following abbreviations will be used in the succeeding chapters, in addition to those given at the commencement of Chap. III. :—

D.P. = Directing plane.
A.M.P. = Accidental measuring point.
H.T. = Horizontal trace.
V.T. = Vertical trace.
C.V.L. = Centre of vanishing line.

PROBLEM 57

Determine the V.L. *for planes at right angles to the* P.P., *and inclined to the* G.P. *at an angle of* 30° *on the right. Let this be the* V.L. *for the ends of an equilateral triangular prism,* 1′ *long, with sides* 4′ *wide; the lower left-hand corner of the prism to rest on the* G.P., 4′ *from the* P.P., *and* 1′ *to the spectator's right, the right-hand side of the prism to be inclined to the* P.P. *at an angle of* 65°.
Distance of P.P. = 8′. *Height of eye* = 4′. *Scale* $\frac{1}{4}'' = 1'$.
Fig. 122.

Determine the G.L., H.L., C.V., and perspective scale, as in Prob. 5. Also the $\frac{1}{2}$ D.P., and the given point of the prism (a), as in Prob. 10.

Draw through C.V. a line making with H.L. an angle of 30° on the right. This is the vanishing line required.

Draw a vertical line from C.V., 8′ long, to find S (the station point).

Draw a line from C.V. at right angles to V.L., and with C.V. as centre, and the distance of S as radius, draw an arc to

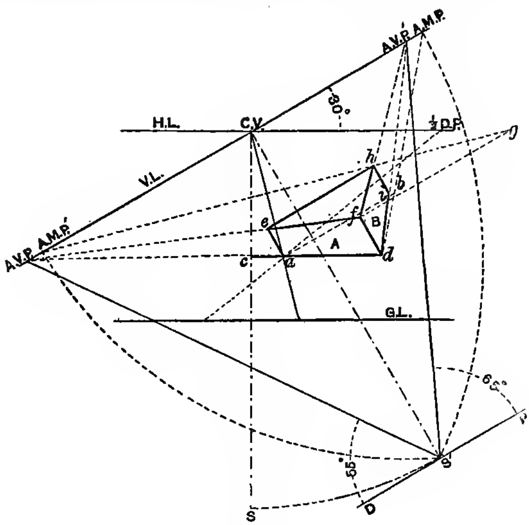


Fig. 122

meet this line in S′. Draw the line D.P. (directing plane) at right angles to this line [69].

As the prism is to be equilateral, and the right-hand side is inclined at an angle of 65° with the P.P., the left-hand side must recede at an angle of 55° (60° + 65° + 55° = 180°). Draw lines from S′ making these angles with D.P., and produce them to meet the V.L. in the points A.V.P. and A.V.P.′, the accidental vanishing points required.

With these points as centres, and the distance of each from S′ as radius, draw arcs to meet the V.L. in the points A.M.P. and A.M.P.′ These are the accidental measuring points required.

Draw a horizontal line through *a* to find the perspective

foot *ac*. Draw a line from *a*, parallel to V.L. and four times *ac* in length, to find *b*. Draw a line from A.M.P. through *b* to meet a line drawn from A.V.P. through *a* in *d*. As the ends of the prism are at right angles to the P.P., its parallel edges must be parallel to the P.P.

Draw the line *ae* at right angles to *ab*, and equal in length to *ac*. Draw a line from A.V.P. through *e* to meet a line from *d*, parallel to *ae*, in *f*.

Draw lines from the points *d* and *f* to A.V.P.′ Draw a line from *f*, parallel to V.L. and four times *fd* in length, to find *g*. Draw a line from *g* to A.M.P.′ to meet the receding line from *f* to A.V.P.′ in *h*. Draw a line from *h* parallel to *fd* to meet the receding line from *d* in *i*. Join *eh*.

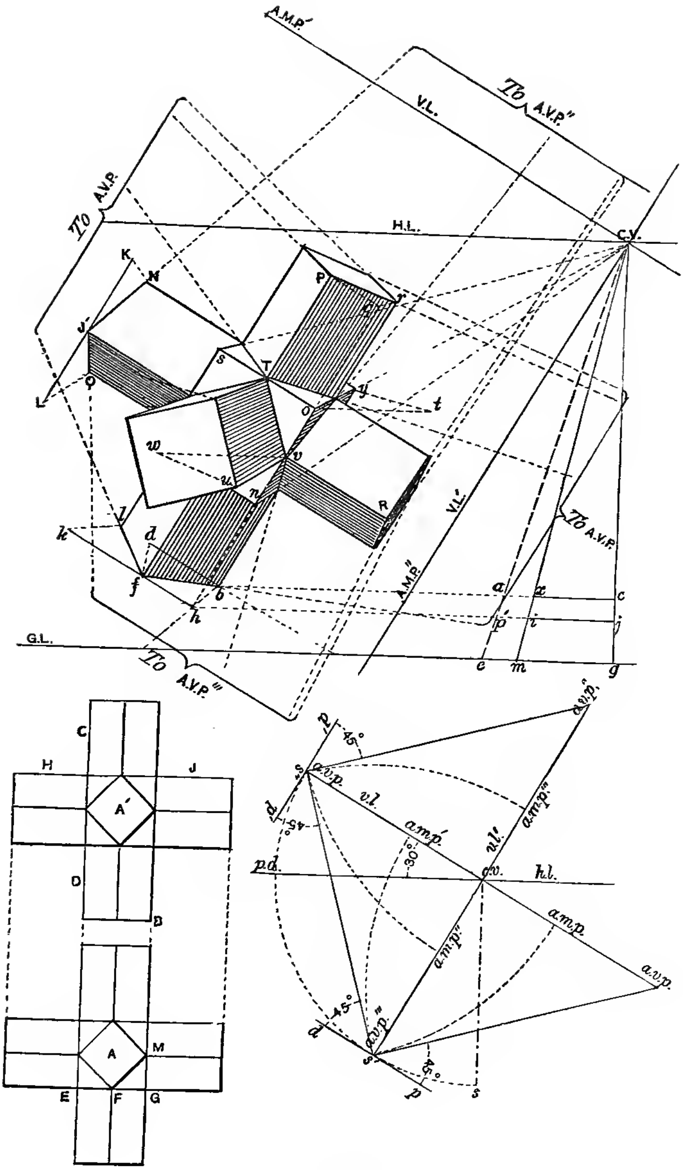
This problem illustrates the use of a V.L. for inclined planes. The sides A and B of the prism are also inclined planes, and their vanishing lines would be drawn through A.V.P. and A.V.P.′ at right angles to V.L.

Problem 58

Let A *and* A′ *be the plan and elevation of a double cross drawn to a scale of* $\frac{1}{4}'' = 1'$. *Draw this cross in perspective, with the point* B *resting on the* G.P. 2′ *from the* P.P., *and* 5′ *to the spectator's left. The ends of the arms* C *and* D *are to be in planes at right angles to the* P.P., *and inclined to the* G.P. *at an angle of* 30° *to the left. The arms* H *and* J *are to be parallel to the* P.P.

Distance of P.P. = 11′. *Height of eye* = $4\frac{1}{2}'$. *Scale* $\frac{1}{2}'' = 1'$. Fig. 123.

To show the working points to this scale would require a sheet of paper about 11″ square, so to overcome this difficulty we will show how they are found in a supplementary drawing to a reduced scale, viz. $\frac{1}{10}'' = 1'$. This method has many advantages, the principal one being that the working lines, points, etc., are not likely to be confused with the lines necessary for the



A.M.P.'
V.L.
To A.V.P.''
To A.V.P.
H.L.
C.V.
K
N
P
J'
O
L
S
T
y
t
o
w
v
u
n
R
V.L.'
To A.V.P.
k
l
d
f
b
h
a
x
c
p'
i
j
A.M.P.''
G.L.
e
m
g
To A.V.P.'''
C
H
J
A'
D
B
A
M
E
F
G
45°
a.v.p.
vl.
a.m.p.'
a.m.p.''
vl.'
p.d.
30°
c.v.
h.l.
a.m.p.
a.v.p.''

Fig. 123

construction. Italics are used for the names of the different working points, etc., in the supplementary drawing, and capitals for the corresponding points, etc., in the larger drawing.

Draw a horizontal line *h.l.* for the horizon line, and assume any point *c.v.* upon it for the centre of vision.

Draw a line from *c.v.*, at right angles to *h.l.*, 11′ long, to find *s*, the station point.

Draw a line from *c.v.*, making an angle of 30° with *h.l.*, and produce the line for the *v.l.* Draw a line from *c.v.*, at right angles to *v.l.*, 11′ long, to find *s′*, and draw the line *dp* at right angles to it for the directing plane [69].

It will be seen from the plan of the cross that the receding sides of the arms C and D are at an angle of 45° with the P.P. Draw lines making these angles at *s′* with *dp*, and produce them to meet *v.l.* in the points *a.v.p.* and *a.v.p.′* Determine the *a.m.p.*'s corresponding to these points, as in the preceding problem.

We must now find the V.L. for planes containing the ends of the arms H and J. As these planes are at right angles to the planes forming the ends of the arms C and D, the V.L.'s must also be at right angles to each other; so draw a line through *c.v.* at right angles to *v.l.* to find *v.l.′* This is the V.L. for a plane inclined at an angle of 60° with the G.P., and at right angles to the P.P. To determine the position of the station point (*s″*) and directing plane (*d′p′*) in reference to this V.L., draw a line 11′ long from *c.v.*, at right angles to *v.l.′* This coincides with *v.l.* The receding sides of the arms H and J are also inclined at angles of 45° with the P.P.; so set off these angles at *s″* with *d′p′*, and proceed as before to find the *a.v.p.*'s and *a.m.p.*'s.

Note.—Any convenient scale may be chosen for this supplementary drawing, but it is advisable to select one that is an aliquot part of the larger scale.

We will now commence the larger drawing.

Find the necessary working lines, points, etc., for finding the point *b* (Prob. 5).

Draw the two V.L.'s through C.V. at the same angles as shown in the supplementary drawing. As the scale of this drawing is five times that of the smaller drawing, repeat the distance of each working point from *c.v.* five times from C.V. on its corresponding V.L., *e.g.* the distance between C.V. and A.M.P.′ is five times the distance between *c.v.* and *a.m.p.*′ A.M.P.″ is determined in a similar manner.

We will now construct two perspective scales, viz. one from the points *m*, *g*, to represent the width (FM) of the side of each arm, and the other from the points *e*, *g*, to represent the width (EG) of the cube forming the centre of the cross.

Draw a line *bd* parallel to V.L., and equal in length to *xc*. Draw a line from A.M.P.′ through *d* to meet a line drawn from A.V.P.′ through *b* in *f*. Draw a line through *f* parallel to V.L. to meet a line from C.V. through *b* in *h*. Draw a horizontal line from *h* through the perspective scales. Draw a line from *h* parallel to the V.L.′, and mark off upon it the following lengths from the perspective scales: viz. $hn = 1\frac{1}{2}$ times ij, $no = p'j$, and $oq = hn$. Draw *os* at right angles to *no* and of the same length. Join *n*, *o*, *s*, to C.V. Draw a horizontal line $ot = no$, and draw a line from *t* to P.D. to meet the receding line from *o* to C.V. in *y*. Complete the cube forming the centre of the cross, and bisect each of its front edges. Draw lines from these points parallel to their corresponding V.L.'s. Draw a line from *q*, parallel to *os*, to find P. Then PT will give us the length of the front edge of each arm that is parallel to the P.P. Set off these lengths for the respective edges, and draw lines from their extremities towards their corresponding A.V.P.'s, as shown.

Produce *hf* and set off $fk = ij$. Join *k* to A.M.P. to meet the receding line from *f* in *l*. The points O and N are found in a similar manner.

To determine the length of the front arm, draw a horizontal line *vw* equal in length to TP. Draw a line from P.D. through *w* to meet a line from C.V. through *v* in *u*. Complete the cross, as shown.

Problem 59

Draw an equilateral triangular prism 6′ long, and with each face 3′ wide, lying on one of its longer edges, with one of its faces inclined to the G.P. *at an angle of* 45°. *Its longer edges are to recede to the left at an angle of* 50° *with the* P.P., *and the nearer end of the edge on the* G.P. *is to be* 5′ *from the* P.P. *and* 4′ *to the spectator's right.*

Distance from P.P. = 7′. *Height of eye* = 4′. *Scale* $\frac{1}{4}'' = 1'$. Fig. 124.

As the edges of the prism recede to the left at an angle of

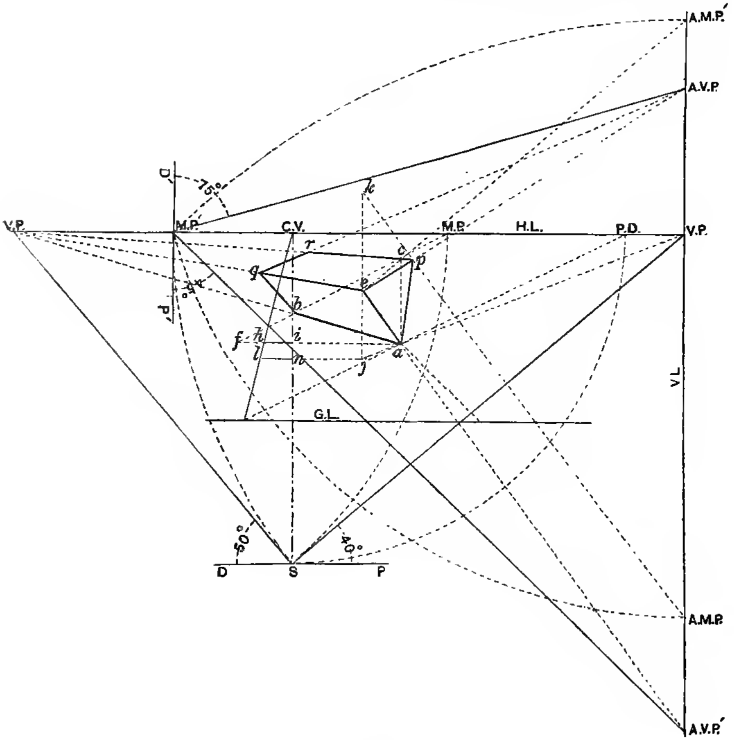


Fig. 124

50° with the P.P., the vertical planes containing its ends must

recede to the right at an angle of 40° ; and as the V.L. for such planes must pass through the V.P. for horizontal lines in the plane [59], first find the V.P.'s (V.P. and V.P.′) for horizontal lines at these angles, as well as the point *a* (Prob. 12).

The planes containing the ends of the prism being vertical planes inclined to the P.P. at an angle of 40°, a vertical line drawn through V.P.′ gives the V.L. for those planes [59].

The distance of M.P.′ from V.P.′ being the same as the distance of S from V.P.′, a vertical line drawn through M.P.′ gives the position of the directing plane (D′P′) with reference to the V.L.

If one side of the prism is inclined to the G.P. at an angle of 45°, it must also be inclined to the vertical at the same angle (45° + 45° = 90°); and as the prism is equilateral and triangular, another side must be inclined to the vertical at an angle of 75° (60° + 45° + 75° = 180°).

Set off these angles at M.P.′ with the line D′P′, and produce the lines to meet V.L. in the points A.V.P. and A.V.P.′ With each of these points as a centre, and the distance of M.P.′ as a radius, draw arcs to meet V.L. in the points A.M.P. and A.M.P.′

Draw a line at *a* parallel to V.L., and three times *hi* in length, to find *c*. Draw a line from A.M.P.′ through *c* to meet a line drawn from A.V.P.′ through *a* in *e*.

Draw a line from V.P.′ through *a* to meet a vertical line from *e* in *j*. Draw a horizontal line from *j* to find the perspective foot *ln*. Draw *ek* = three times *ln*, parallel to V.L., and draw a line from *k* to A.M.P. to meet a line from *e* to A.V.P. in *p*. Join *pa*.

Join *e*, *p*, and *a* to V.P. Set off *af* = six times *hi*, and join *f* to M.P. to meet the receding line from *a* in *b*. Draw a line from A.V.P.′ through *b* to find *q*. Join *q* to A.V.P. to find *r*. This completes the prism.

Problem 60

Let A *and* B *be the front and side elevations of an object drawn to a scale of* $\frac{1}{5}'' = 1'$. *Draw this object in perspective, with the front in a vertical plane, inclined to the* P.P. *at an angle of* 40° *on the left; the side* C.D. *to be inclined to the* G.P. *at an angle of* 42°; *the corner* C *to be* 3′ *from the* P.P. *and* 2′ *to spectator's left.*

Distance of P.P. = 9′. *Height of eye* = 4′. *Scale* $\frac{1}{2}'' = 1'$. Fig. 125.

The lines forming the perspective representation of this

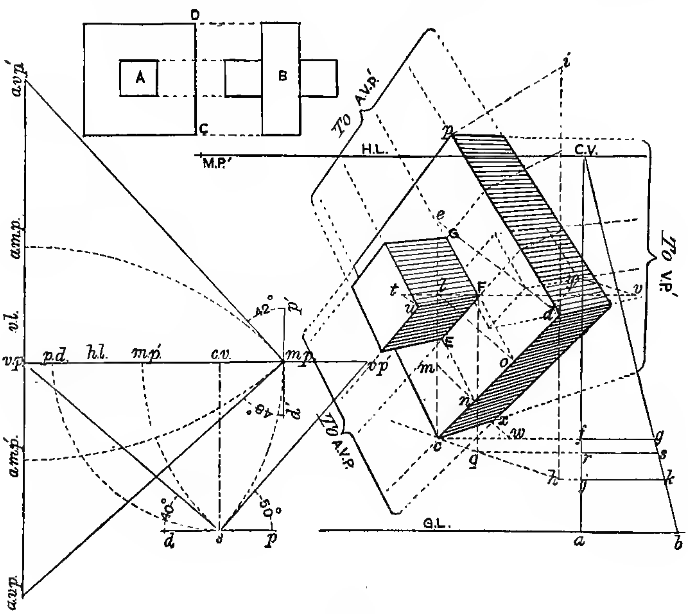


Fig. 125

object are drawn to V.P.'s on a V.L. for vertical planes inclined to the P.P. at an angle of 40° on the left, and to a V.P. for lines at right angles to these planes.

First determine the working points, etc., by means of a supplementary drawing, to a scale of $\frac{1}{10}'' = 1'$.

If the front of the object is inclined to the P.P. at an angle of 40° on the left, the edges receding to the right must be inclined to the P.P. at an angle of 50°.

Determine the V.P.'s, M.P.'s, etc., for these angles (Prob. 12).

Draw a vertical line through *v.p.* for the *v.l.*; also a vertical line through *m.p.* for the directing plane (Prob. 59). As the side CD is inclined to the G.P. at an angle of 42°, its angle of inclination to the vertical must be 48° (90° − 42° = 48°), and that of each of the ascending edges must be 42°. Draw lines making these angles at *m.p.* with *d.'p.'*, and produce them to meet *v.l.* in the points *a.v.p.* and *a.v.p.'* Determine the *a.m.p.*'s (Prob. 59).

To commence the larger drawing, find the necessary working points to determine the point *c* (Prob. 5). As the dimensions of the object are multiples of a foot, construct a perspective scale for one foot from the points *a* and *b*.

As the scale of this drawing is five times that of the supplementary drawing, increase the distances between the working points to this proportion, starting from C.V.

Draw a vertical line at *c*, and mark off upon it at *m*, *l*, and *e* three perspective feet, each equal to *fg*. Draw lines from A.M.P. through these points to meet a line drawn from A.V.P. in the points *n*, *o*, *d*.

Draw a line from V.P. through *c* to meet a vertical line from *d* in *h*. Draw a horizontal line from *h* to determine the perspective foot *jk*. Draw a vertical line at *d*, three times *jk* in height, and draw lines from each foot to A.M.P.', meeting a line drawn from *d* to A.V.P.' in the points shown on the line *dp*. Draw lines from these points to A.V.P., intersecting lines drawn to A.V.P.' from the points *n*, *o*, in the points E, F, G.

Draw lines from V.P.' through the points E, F, G. Draw a vertical line from F to meet the line *ch* in *q*. Draw a horizontal line from *q* to determine the perspective foot *rs*. Draw a

horizontal line $Ft = rs$. Draw a line from M.P.′ through t to meet the line from V.P.′ through F in u. Draw lines from the points u, p, c, to their respective A.V.P.'s, and complete the front of the object, as shown. The point y is found by drawing $Fv =$ twice rs and joining v to M.P.′

Draw lines from the points c, d, p to V.P.′ Make $cw = fg$, and join w to M.P.′ to meet the receding line from c in x. Complete the object, as shown.

Problem 61

Let A *and* A′ *be the plan and elevation of an object drawn to a scale of* $\frac{1}{6}'' = 1'$. *Draw this object in perspective with its under side inclined to the* G.P. *at an angle of* 35°; *the farthest lower corner of the object to lie on the* G.P. 6′ *from the* P.P. *and* 1′ *to the spectator's right. The longest edges are to form angles with the* P.P. *of* 40° *on the right and* 50° *on the left in a descending plane.*

Distance of P.P. = 10′. *Height of eye* = 5′. *Scale* $\frac{1}{3}'' = 1'$ Fig. 126.

The lines forming the perspective representation of this object are drawn to V.P.'s on a V.L. for descending planes, and to a V.P. for lines at right angles to these planes.

First make a supplementary drawing of the working points, etc., to a scale of $\frac{1}{12}'' = 1'$, as follows :—

Draw *h.l.*, and assume any point on it for *c.v.* Find *p.d.* As the distance of *p.d.* from *c.v.* is the same as that of the spectator from the P.P., draw a vertical line through *c.v.* and assume it to be the edge of the P.P. Also draw a vertical line through *p.d.* for the edge of the D.P.

As the face of the object is inclined to the G.P. at an angle of 35°, its angle of inclination to the P.P. must be 55° (90° − 35° = 55°); and the shorter edges of the object must be inclined to the P.P. at an angle of 35°. Draw lines making

these angles at *p.d.* with *d.p.*, and produce them to meet the vertical line through *c.v.* in the points marked *c.v.l.* (centre of vanishing line) and *a.v.p.*″ Then *c.v.l.* must be the level of the

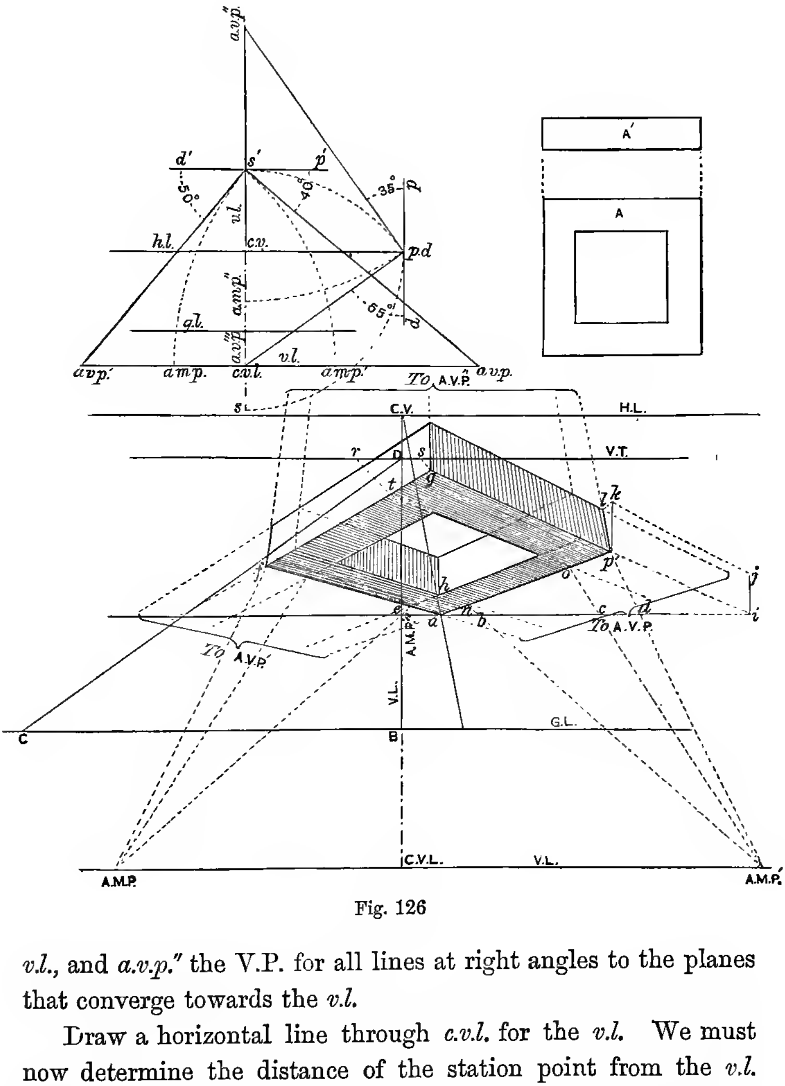


Fig. 126

v.l., and *a.v.p.*″ the V.P. for all lines at right angles to the planes that converge towards the *v.l.*

Draw a horizontal line through *c.v.l.* for the *v.l.* We must now determine the distance of the station point from the *v.l.* With *c.v.l.* as centre and its distance from *p.d.* as radius, draw

an arc to meet the vertical line through *c.v.* in s'. Draw a horizontal line *d.'p.'* through s' for the edge of the directing plane. Draw lines making the given angles at s' with *d.'p.'*, and produce them to meet *v.l.* for the *a.v.p.*'s. Determine the corresponding *a.m.p.*'s (Prob. 59).

Set off the working points, etc., for the larger drawing to four times the scale of the supplementary drawing, starting from C.V.

Determine the necessary working lines, points, etc., to find the point a in the larger drawing; also a perspective scale for one foot (Prob. 5).

Draw a horizontal line through a, and determine the perspective foot ea. Set off from a, $ab = ea$, $bc = 3$ times ea, and $cd = ea$. Draw lines from A.M.P.′ through the points b, c, and d to meet a line drawn from A.V.P.′ through a in the points n, o, and p'. Find corresponding points on the adjacent side of the object in a similar manner, and complete the under side of the object, as shown.

Draw lines from the points f, g, p', and h to A.V.P.″ Produce the line gp' to meet the horizontal line through a in i. Draw a vertical line $ij = ea$. Draw a line from A.V.P. through j to meet a vertical line at p' in k. Join k to A.M.P.″, meeting the line from p' to A.V.P.″ in l. Complete the drawing of the object, as shown.

The receding widths of this object could also be measured from the V.T. (vertical trace) of the plane containing the under side of the object, *e.g.*—

Set off on the G.L., $BC = 6'$ (the distance of the point a from P.P.). Draw a line at C making an angle of 35° with G.L., and meeting the vertical line at B in D. Draw a horizontal line through D for the V.T. Draw a line from A.M.P.′ through g to meet V.T. in s. Set off $sr = 1'$, and draw r to A.M.P.′ to meet the receding line from g in t. The other widths could be measured in a similar manner.

PROBLEM 62

Let A *and* A′ *be the plan and elevation of a double cross, drawn to a scale of* $\frac{1}{8}'' = 1'$. *Draw this cross in perspective, with the corner* F *resting on the* G.P., $3\frac{1}{4}'$ *from the* P.P. *and* 3′ *to spectator's right; the upper surface of the arms* B, E, C, D *to be in an ascending plane inclined to the* G.P. *at an angle of* 42°; *the line* BC *to form an angle of* 37°, *and the line* ED *an angle of* 53° *with the* P.P. *in the ascending plane.*

Distance of P.P. $= 9\frac{1}{2}'$. *Height of eye* $= 4'$. *Scale* $\frac{1}{2}'' = 1'$. Fig. 127.

The lines forming the perspective representation of this object are drawn to V.P.'s on a V.L. for ascending planes, and to a V.P. for lines at right angles to these planes.

Make a supplementary drawing of the working points, etc., to a scale of $\frac{1}{8}'' = 1'$. The construction for this is very similar to that for the preceding problem, the only difference being that the *v.l.* for the ascending plane is drawn through the higher *a.v.p.* instead of the lower, the preceding illustration being for a descending plane.

Find the G.L., H.L., and perspective scale for one foot, to a scale of $\frac{1}{2}'' = 1'$. Set off 3′ to the right of the spectator to find *g*. Set off $3\frac{1}{4}'$ to the left of *g*, on the G.L., to find *h*. Draw a line from *h* to P.D., meeting a line from *g* to C.V. in *f*.

Draw a horizontal line through *f* to find the perspective foot *ij*. Set off $fr = ij$, *rn* and *fj* each equal to twice *ij*. Draw a line from A.M.P.‴ through *j* to meet a line from A.V.P.‴ through *f* in *u*. Draw lines from *r* and *n* to A.M.P.‴, meeting the receding line from *f* in the points *s* and *t*. Find the points *p′*, *o*, *q* in a similar manner.

Draw a vertical line at *f*, and set off *f*G = twice *ij*, GH = *ij*, and HJ = *f*G. Draw lines from A.M.P. through the points

J, H, G to meet a line drawn from A.V.P. through *f* in the points N, L, K.

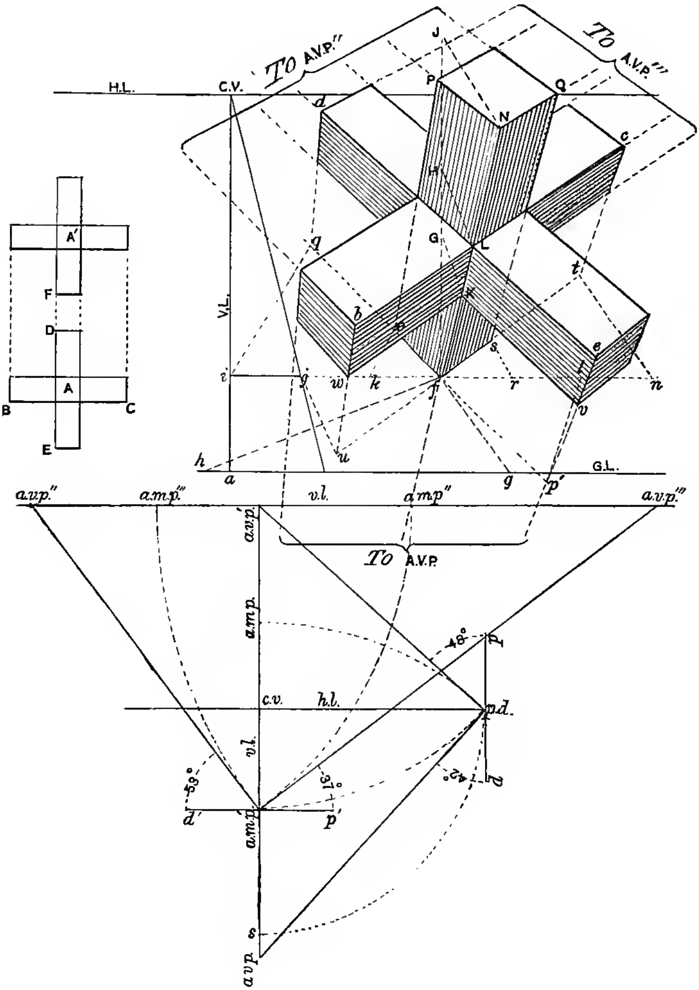


Fig. 127

Draw lines from A.V.P.''' through the points N, L, K to meet lines drawn from A.V.P. through the points *u*, *s*, *t* to determine the points *b*, Q, *c*. Find the points *e*, P, *d* in a

similar manner, and draw lines from them to the several V.P.'s to complete the cross, as shown.

Problem 63

Let A *and* A′ *be the plan and elevation of an object drawn to a scale of* $\frac{1}{8}'' = 1'$. *Draw this object in perspective, with the edge* BC *resting on the* G.P., *and receding to the left at an angle of* 50° *with the* P.P; *the corner* C *to be* 3′ *from the* P.P. *and* 2′ *to the spectator's right; the surface* D *to be in an oblique plane, inclined to the* G.P. *at an angle of* 50°. *Show where the* G.P. *will intersect the object.*

Distance of P.P. = 14′. *Height of eye* = 5′. *Scale* $\frac{3}{8}'' = 1'$. Fig. 128.

The lines forming the perspective representation of this object are drawn to V.P.'s on a V.L. for oblique planes, and to a V.P. for lines at right angles to these planes.

Make a supplementary drawing of the working points, etc., to a scale of $\frac{1}{16}'' = 1'$, in the following manner :—

Draw any horizontal line *h.l.*, and assume any point on it for *c.v.* Draw a line from *c.v.* at right angles to *h.l.*, 14′ long, to find *s.* Draw a horizontal line through *s* for *dp.* Draw a line making an angle of 50° at *s* with *dp*, and produce this to meet *h.l.* in *v.p.* This is the V.P. for the line CB. If the horizontal lines of the object recede to the left at an angle of 50° with the P.P., planes at right angles to these lines will recede to the right at an angle of 40°. Find the V.P. for horizontal lines in these planes, as *v.p.*′ Determine *m.p.*′ Draw a vertical line through *v.p.*′ for the V.L. for vertical planes receding to the right at an angle of 40° with the P.P. As the distance of *m.p.*′ from *v.p.*′ is the same as the distance of the station point from it, draw a vertical line through *m.p.*′ for the D.P., as *d′p′*. As the oblique plane is inclined to the G.P. at an angle of 50°, its angle of inclination with the vertical must be 40° (90° — 50° = 40°). Set

off this angle at *m.p.′* with *d′p′*, and produce the line to meet *v.l.* in *a.v.p.* Join *a.v.p.* and *v.p.* for *v.l.′* This is the V.L. for the oblique plane.

As the oblique plane is inclined to *d′p′* at an angle of 40°,

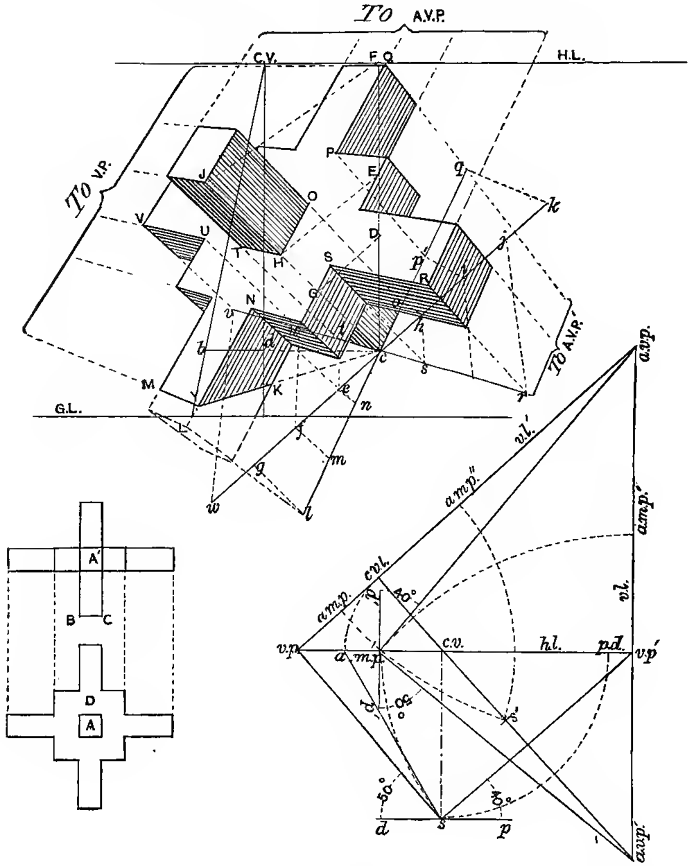


Fig. 128

lines at right angles to the oblique plane must be inclined to *d′p′* at an angle of 50°. Draw a line making this angle at *m.p.′* with *d′p′*, and produce this to meet *v.l.* in *a.v.p.′* This is the V.P. for the descending edges of the object. Find *a.m.p.′* corresponding to *a.v.p.′*

We must now find the distance of the station point from *v.l.*′ Draw a line through *c.v.* at right angles to *v.l.*′, meeting it in *c.v.l.* [87]. With *c.v.* as centre, and its distance from *c.v.l.* as radius, draw an arc to meet *h.l.* in *a*. Join *as*. Then *as* is the generatrix of a cone tangential to the oblique plane [74], and its length is equal to the distance of *s* from *c.v.l.* Set off from *c.v.l.* on the line passing through *c.v.*, a distance equal to *as*, to find *s*′. With *v.p.* and *a.v.p.* as centres, and with their respective distances from *s*′ as radii, draw arcs to meet *v.l.*′ in the points *a.m.p.* and *a.m.p.*″

Find the necessary working points in the larger drawing to determine the point *c*, and construct a perspective scale for 1′ (Prob. 5).

Set off the working points, etc., for the larger drawing to six times the scale of the supplementary drawing, starting at C.V.

Draw a horizontal line through *c* to find the perspective foot *bd*. Draw a line through *c* parallel to V.L.′, and set off upon it, from *c*, the following distances, viz. *ce*, *ef*, *ch*, *hi*, each equal to *bd*; and *ik*, *eg*, each equal to twice *bd*. Draw lines from A.M.P. through the points *e*, *g* to meet a line drawn from A.V.P. through *c* in *n*, *l*. Draw lines from the points *h*, *k* to A.M.P. to meet the receding line from *c* in the points *o*, *p*′, *q*. Determine similar points on a line drawn from V.P. through *c* by drawing lines from A.M.P.″ through the points *j*, *h*, *e*, *f*, *w* to meet it in the points *r*, *s*, *t*, *u*, *v*. These points, being on a horizontal line, could have been found by means of the M.P. corresponding to V.P., on the H.L.

Draw a vertical line at *c*, and set off *c*D = twice *bd*, DE = *bd*, and EF = *c*D. Draw lines from A.M.P.′ through F, E, D to meet a line drawn from A.V.P.′ through *c* in the points J, H, G.

Draw lines from A.V.P.′ through the points *l*, *n*, *o*, *p*′, *q* to meet lines drawn from A.V.P. through the point H in the points L, N, O, P, Q. Draw lines also from A.V.P.′ through the points *r*, *s*, *t*, *u*, *v* to meet a line drawn through H from

V.P., in the points R, S, T, U, V. Complete the drawing, as shown.

To determine where the G.P. intersects the lowest arm of the object, draw a line from V.P.′ through c to meet the line NL in Y. Draw a line from Y to V.P. to find the point M.

Problem 64

1. *Draw in perspective a regular octagon of* $1\frac{1}{2}'$ *side, with one of its sides resting on the* G.P., *and inclined to the* P.P. *at an angle of* 50° *to the spectator's right; two of its sides to be inclined to the* G.P. *at an angle of* 46°, *and to be contained by vertical planes inclined to the* P.P. *at an angle of* 40° *to the spectator's left; the nearer end of the lowest side to be* 2′ *from the* P.P. *and* 2′ *to the spectator's left.*
2. *Inscribe a circle in the octagon, and let it form the base of a right cone, with an axis* 6′ *long.*
3. *Draw a circle of the same diameter as the base of the cone, and parallel to it, so as to form a right cylinder,* 6′ *long.*

Distance of P.P. = 9′. *Height of eye* = 5′. *Scale* $\frac{1}{3}'' = 1'$. Fig. 129.

Make a supplementary drawing for finding the necessary V.P.'s, etc., for these angles, as shown, and as already described in the preceding prob., to a scale of $\frac{1}{18}'' = 1'$.

Set off the distances between these points to six times this scale in the larger drawing, commencing at C.V.

Construct a perspective scale, and find the position of the point a (Prob. 5).

1. Draw a horizontal line through a, and in any convenient position on this line, and with the perspective foot at this distance, construct a regular octagon of $1\frac{1}{2}'$ side, as shown. Set off from a, on the horizontal line drawn through it, the points j', b', k', at distances equal to those between the corresponding

points J, A, B, K. Draw a line from M.P.′ through j' to meet a line drawn from V.P.′ through a in j. Also draw lines from b', k' to M.P.′ to meet the same line in b, k. Draw a vertical line at a, and mark off the points l', n', f' at distances equal to those marked on A, L, N, F. Draw lines from A.M.P. through these points to meet a line drawn from A.V.P. through a in the points l, n, f. Draw lines through these points to V.P.′, intersecting lines drawn from A.V.P. through the points

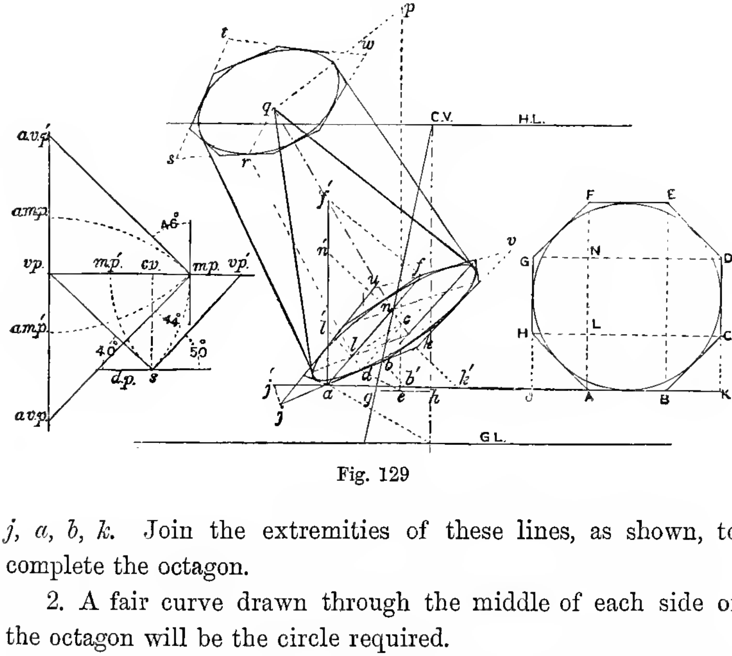


Fig. 129

j, a, b, k. Join the extremities of these lines, as shown, to complete the octagon.

2. A fair curve drawn through the middle of each side of the octagon will be the circle required.

Note.—This circle could be drawn by finding eight points in the circumference, as described in Prob. 25, but it can be more accurately drawn to a circumscribing octagon, as here shown, than through eight points, as described in the elementary stage.

Find the centre of the circle (c) by drawing any two diagonals to the octagon through opposite angles, and draw a line from c to A.V.P.′ for the axis of the cone. Draw a vertical line from c.

To find the size of a perspective foot at the distance of c, proceed in the following manner :—Draw a line from c to A.V.P., meeting ab in d. Draw a line from V.P. through d to meet a vertical line from c in e. Draw a horizontal line through e to find gh, the perspective foot required. Set off from c on the vertical line cp = six times gh. Draw a line from p to A.M.P.′ to meet the axis in q. Draw lines from q, forming tangents to the base of the cone.

3. Draw a line from q to A.V.P. to meet a line drawn from d to A.V.P.′ in r. Draw a line from V.P.′ through r to meet a line drawn from j to A.V.P.′ in s. Draw a line from A.V.P. through s to meet a line drawn from u to A.V.P.′ in t. Draw a line from t to V.P.′ to meet a line drawn from v to A.V.P.′ in w. Draw lines from the respective angles of the octagon to A.V.P.′, to meet these lines, and join the corresponding angles, as shown, to form another octagon; inscribe a circle in this octagon, as already described. Draw tangents to each circle to complete the cylinder.

This problem also shows how a right pyramid in an oblique position could be constructed by joining the point q with the angles of the base; also how a regular prism in an oblique position could be constructed by joining the corresponding angles of the two ends.

Curves on an oblique plane are drawn by first intersecting the given curve by lines at right angles, forming equal squares, as shown in Prob. 34, and then constructing similar squares on the oblique plane, and drawing the curve through the corresponding intersections, as described in Prob. 34.

In Articles 53 and 66 we have shown how the positions of lines inclined to both the P.P. and G.P. may be determined, by assuming them to be contained by a plane at a given angle with either the P.P. or G.P.; we will now show how such a line may be drawn in perspective from the given angles of inclination of the line itself with the P.P. and G.P.

PROBLEM 65

Draw in perspective a line 6′ *long, receding towards the right, and inclined to the* P.P. *at an angle of* 48°, *and to the* G.P. *at an angle of* 34°; *the lower end of the line to meet the* G.P. 3′ *from the* P.P. *and* 3′ *to the spectator's right.*
Distance of P.P. = 10′. *Height of eye* = 5′. *Scale* $\frac{1}{8}'' = 1'$. Fig. 130.

Find the necessary working points, etc., and determine the position of the point *a* (Prob. 5).

Draw DP through S parallel to H.L. Draw a line making

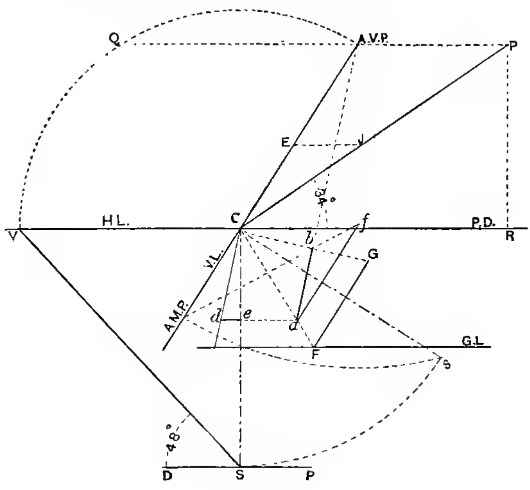


Fig. 130

an angle of 48° at S with DP, and produce the line to meet H.L. in V. Then SV is the generatrix of a cone [52]. With C as centre, and radius CV, draw an arc to represent a portion of the base of this cone on the P.P.

Draw a line at C, making, with H.L., an angle of 34°, and produce the line to P′, making CP′ = SV. It is evident that P′R is the height of one end of the generatrix above the axis of the cone, when inclined at an angle of 34° with the G.P.

Draw a horizontal line from P′ to meet the base of the cone in A.V.P. Join C to A.V.P. by a line which represents the generatrix of the cone projected on the P.P., and is therefore the vanishing parallel for the given line [13]. Consequently A.V.P. is its V.P.

Note.—If the line were receding towards the left, Q would be its A.V.P.

Set off from C, on CP′, 6′ to find point J. Draw a horizontal line from J to meet the line from C to A.V.P. in E. Then CE is the picture length of the given line (p. 109). Produce a line from C through *a* to meet G.L. in F. Draw a line from F parallel and equal in length to CE, as FG. Join G to C, meeting a line drawn from *a* to A.V.P. in *b*. Then *ab* is the line required.

This line could also be measured by an A.M.P. corresponding to A.V.P., *e.g.* we know from Article 53 that a line inclined to both the P.P. and the G.P. can be contained by a plane at right angles to the P.P. Then produce the line from A.V.P. to C to get the V.L. for such a plane. Draw C*s* at right angles to V.L. and equal in length to CS. With A.V.P. as centre, and the distance of *s* as radius, draw an arc to meet V.L. in A.M.P.

Draw a horizontal line from *a* to find the perspective foot *de*. Draw a line from *a* parallel to V.L., and six times *de* in length, to find *f*. Draw a line from *f* to A.M.P. to meet a line from *a* to A.V.P. in *b*.

In Article 80 we have shown how the V.L. for an oblique plane may be determined by joining the V.P.'s for lines contained by the oblique plane; we will now show how the V.L. for an oblique plane may be determined from the angles the oblique plane itself forms with the P.P. and the G.P.; but, before doing so, we will study Fig. 131, which is a perspective sketch showing an oblique plane P intersected by a line CA at right angles to it.

If we project the line CA on a horizontal plane we make a plan of it, as C*a*; and if we project it on a vertical plane,

the P.P. for instance, we make an elevation of it, as Ca'. Now the line Sg, which is the H.T. of the plane P, must be at right angles to Ca; and the line gf or V.L., in which the plane P meets the P.P., must be at right angles to Ca'. This will be seen from the following:—

If the angles of inclination of the plane P with the G.P. and the P.P. were given, we could determine the angles the line CA would form with each plane, *e.g.* let the plane P be inclined to the G.P. at an angle of 60° and to the P.P. at an

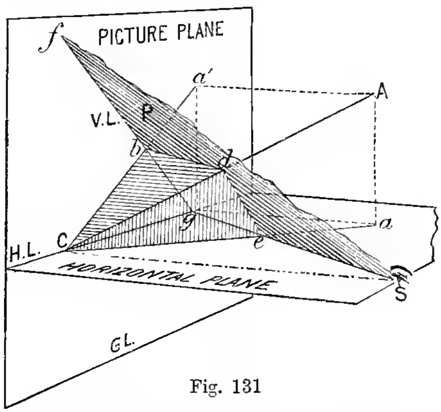


Fig. 131

angle of 50°. Assume Cde to be a vertical plane; this plane is at right angles to the plane P, consequently Ced is the angle at which P is inclined to a horizontal plane, or to the G.P. [86]. Since the angle $Ced = 60°$, the angle eCd must be 30°. Next, since Cbd is a plane at right angles to plane P, for we have assumed Ca' to be the projection of CA upon the P.P., the angle $Cbd = 50°$, which is the angle of inclination of the plane P to the P.P. [86]. Also the angle bCd is 40°, which is the angle of inclination of the line CA with the P.P.

If, therefore, the angles of inclination of an oblique plane were given, we should first determine the angles of inclination a line from the C.V., at right angles to the oblique plane, would form with the G.P. and the P.P.; we should then make a plan and elevation of this oblique line, then draw a H.T. of the oblique

plane at right angles to the plan of the line from the station point, and from the point in which this H.T. meets the P.P. we should draw the V.L. of the oblique plane at right angles to the projection of the oblique line on the P.P., as illustrated in Fig. 131.

A method of finding a plane of this character from given data is comprised in the following problem.

Note.—If we assume the line CA to be the generatrix of a cone, and CS its axis, the base of the cone will be turned towards the spectator, and C will be its vertex.

Problem 66

Let B *and* B′ *be the plan and elevation of an octahedron drawn to a scale of* $\frac{1}{8}'' = 1'$. *Draw this object in perspective, with the points* E, F, G, H *in a plane receding towards the right, and inclined to the* G.P. *at an angle of* 55°, *and to the* P.P. *at an angle of* 48°. *The edge* EF *is to be inclined to the* P.P. *at an angle of* 40°; *the point* N *is to be* 6′ *from the* P.P., 2′ *to the spectator's right, and* 8′ *above the* G.P. *Distance of* P.P. = 16′. *Height of eye* = 5′. *Scale* $\frac{1}{4}'' = 1'$. Fig. 132.

Make a supplementary drawing of the working points, etc., to a scale of $\frac{1}{16}'' = 1'$, in the following manner:—

Draw a horizontal line for the *h.l.* and assume any point *c* upon it for the *c.v.* Draw *cs* = 16′ at right angles to *h.l.*

Determine the angles that a line (*c*A) at right angles to the oblique plane will form with the G.P. and the P.P., *e.g.* since the oblique plane is inclined to the G.P. at an angle of 55°, the line must be inclined to the G.P. at an angle of 35°; and since the oblique plane is inclined to the P.P. at an angle of 48°, the line must be inclined to the P.P. at an angle of 42°.

Draw a line from *c*, making, with *h.l.*, an angle of 42°, and produce the line to meet the *d.p.* in *g*. This is the plan of a generatrix of a cone, the point *c* being its vertex. With *c* as

centre, and a radius equal to *sg*, draw an arc. This is a portion

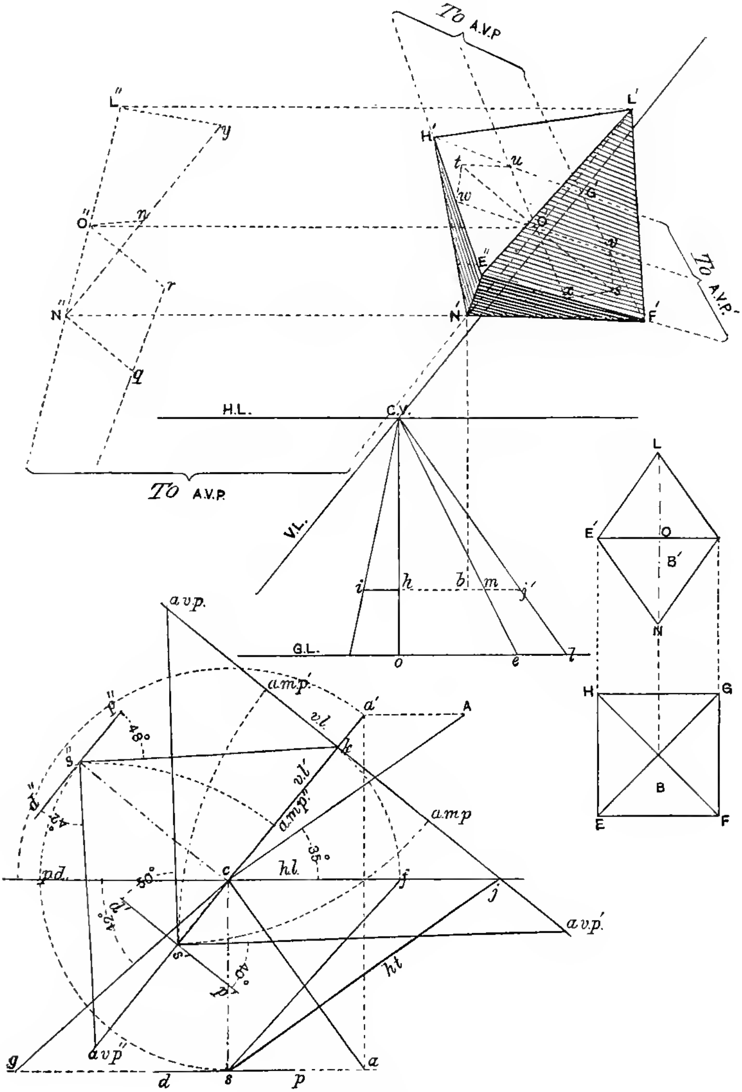


Fig. 132

of the base of the cone projected on the P.P. Draw a line at *c*,

making, with *h.l.*, an angle of 35°, and produce this line till its length is equal to *cg*, to find A. Draw a horizontal line from A to meet the base of the cone in *a'*. Join *a'c*. Draw a vertical line from *a'* to meet *dp* in *a*. Join *ca*. Then *ca* is the plan, and *ca'* the elevation of a line at right angles to the oblique plane (see Fig. 131).

Draw a line from *s* at right angles to *ca* to meet *h.l.* in *j*. This is the *h.t.* of the oblique plane on a horizontal plane at the level of the spectator's eye. Draw a line through *j* at right angles to *ca'*. This is the *v.l.* of the oblique plane.

The line *ca'* intersects the *v.l.* in *k*, consequently *k* is the *c.v.l.* We must now find the distance of *s* from *k*. With *c* as centre, and radius *ck*, draw an arc to meet *h.l.* in *f*. Join *fs*, which is the distance required. (The angle *asf* is the angle the oblique plane forms with the P.P.)

Set off from *k*, on *kc* produced, *ks'* = *fs*. Draw a line *d'p'* through *s'* at right angles to *ks'*. Draw lines from *s'*, making, with *d'p'*, angles of 40° and 50° on the right and left respectively, and produce these lines to meet *v.l.* in the points *a.v.p.'* and *a.v.p.* These are the vanishing points for the edges EF and EH. With these points as centres, and their distances from *s'* as radii, draw arcs to meet *v.l.* in the points *a.m.p.* and *a.m.p.'* These are their corresponding measuring points.

The axis NL of the solid is at right angles to the plane containing the points E, F, G, H. We must now find the V.L. for a plane containing this line, which is found by assuming a plane, at right angles to the oblique plane, passing through the station point. We know that *c* is the projection of the station point on the P.P., therefore a line *ks'* drawn through *c*, at right angles to *v.l.*, is the V.L. (*v.l.'*) required.

Draw a line *cs''* at right angles to *v.l.'*, and equal in length to *cs*. Draw the line *d''p''* at right angles to *cs''*. Join *s''k*. Then the angle *p''s''k* is the angle the oblique plane forms with the P.P. Draw a line from *s''* at right angles to *s''k* to meet *v.l.'* in *a.v.p.''* This is the V.P. for the axis NL.

With *a.v.p.*″ as centre, and its distance from *s*″ as radius, draw an arc to meet *v.l.*′ in *a.m.p.*″

Find the necessary working points, etc., in the larger drawing to determine the point N′ (Prob. 5).

Set off the working points, etc., for the larger drawing to four times the scale of the supplementary drawing, starting from C.V.

Set off from *o* on the G.L., *oe* = twice OE′, also *ol* = twice OL, and join *e* and *l* to C.V. These lines form perspective scales for the lengths of OE′ and OL.

Draw a line from A.V.P.″ through N′. As this line is too near the V.L. of the plane containing it to be measured with any degree of accuracy, we will proceed to measure it in the following manner:—Draw a horizontal line from N′, and assume any point N″ upon it. Draw a line from A.V.P.″ through N″. Draw a line from N″ parallel to V.L.′, and set off N″*n* and *ny*, each equal to *hj*′ in the perspective scale. Draw lines from A.M.P.″ through *n* and *y* to meet the line from A.V.P.″ through N″ in O″ and L″. Draw horizontal lines from O″ and L″ to meet the line from A.V.P.″ through N′ in the points O′ and L′.

Draw a line from N″ parallel to V.L., and equal in length to *hm* in the perspective scale, to find *q*. Draw a line from A.V.P.″ through *q* to meet a line from O″ parallel to N″*q* in *r*.

Draw a line through O′ parallel to V.L., and set off on it from O′, distances equal to O″*r*, to find the points *s* and *t*. Draw lines from the points *s*, *t* to A.M.P. to meet a line drawn from A.V.P. through O′ in *u* and *x*, also from A.M.P.′ through the points *s*, *t* to meet a line from A.V.P.′ through O′ in the points *v* and *w*.

Draw lines from A.V.P. through *v* and *w* to meet lines drawn from A.V.P.′ through *u* and *x* in the points E′, F′, G′, H′. Join H′, E′, F′ to N′ and L′ to complete the solid.

When a solid contains many lines and planes that are not parallel to each other, the work is very much simplified by

finding in perspective the points forming the extremities of the lines. The regular solids furnish good illustrations of objects of this character. In the two following problems, different methods of finding these points are illustrated.

Problem 67

Let A *and* A′ (Fig. 133) *be the plan and elevation of a dodecahedron*[1] *drawn to a scale of* $\frac{1}{2}'' = 1'$. *Draw this object in perspective, in the position shown, to twice this scale, the lowest point* (*b*) *to be on the* G.P., 3′ *from the* P.P., *and* 2′ *to the spectator's left.*

Distance of P.P. = 8′. *Height of eye* = 3′. Fig. 134.

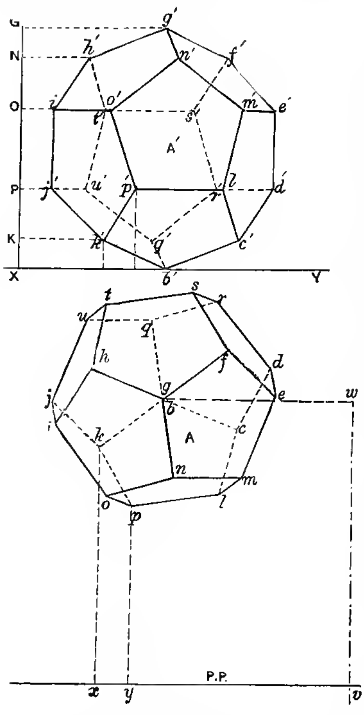


Fig. 133

Draw a horizontal line from *b* (Fig. 133), 2′ long, to find *w*. Draw a line from *w*, 3′ long, at right angles to *bw*, to find *v*. Draw a horizontal line from *v* to represent the position of the P.P. in plan. Draw lines from each of the points in plan at right angles to the P.P. (To prevent confusion, two only, viz. the points *k* and *p*, are taken.)

At any point X on the line XY in the elevation, draw a vertical line, and draw horizontal lines from all the points of the object to meet it.

Draw the G.L., H.L., etc. (Fig. 134), to a scale of $1'' = 1'$, and find point B (Prob. 5).

Draw a line from C.V.

[1] The method of projecting the regular solids, geometrically, is given in Spanton's *Complete Geometrical Course:* Macmillan and Co.

through B to meet G.L. in X. Draw a vertical line at X, and set off upon it the distances XK, KP, etc., equal to twice the corresponding distances on the line XG (Fig. 133). Draw lines from each of these points to C.V. These lines form perspective scales for the heights of all the points of the solid.

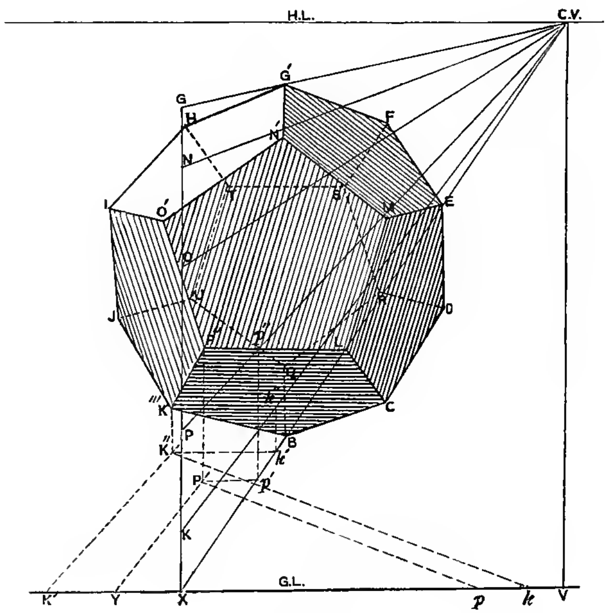


Fig. 134

Set off VY = twice *vy* (Fig. 133), and join Y to C.V. Set off Y*p* = twice *yp* (Fig. 133), and draw a line from *p* towards P.D. to meet the receding line from Y in P′. Draw a horizontal line from P′ to meet the receding line from X in *p*′. Draw a vertical line at *p*′ to meet the receding line from P in *p*″. Draw a vertical line at P′ = the height *p*′*p*″ to find P″. This is one point of the solid. Find all the other points in a similar manner, and join them, as shown.

Problem 68

Let A *and* A′ (Fig. 135) *be the plan and elevation of a icosahedron drawn to a scale of* $\frac{1}{2}'' = 1'$. *Draw this object in perspective to twice this scale, the vertical axis, and the points* c *and* k, *to be in a vertical plane inclined to the* P.P. *at an angle of* 57° *on the right; the lowest point* b *to be* 2′ *from the* P.P. *and* 3′ *to the spectator's left.*
Distance of P.P. = 12′. *Height of eye* = 3′. Fig. 136.

Draw a line from *b* (Fig. 135), making, with *bk*, an angle of

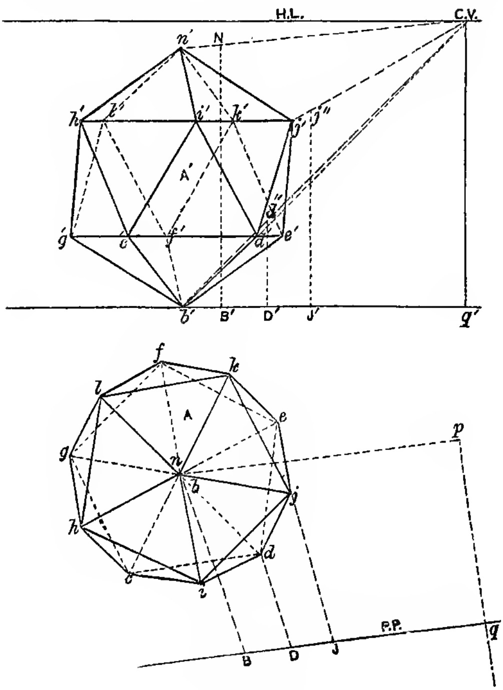


Fig. 135

57°, and produce the line till it is 3′ long to find *p*. Draw a line from *p* at right angles to *bp*, 2′ long, to find *q*. Draw a line from *q* parallel to *bp*, for the position of the P.P. in plan.

Set off from q, on pq produced, 12′, to find the station point. Draw lines from each of the points of the solid, in plan, towards the station point to meet the P.P. To save confusion, only three points are taken, b, d, and j. These lines are projections of the visual rays [9] upon the G.P.

Draw a horizontal line through b' in elevation for the level

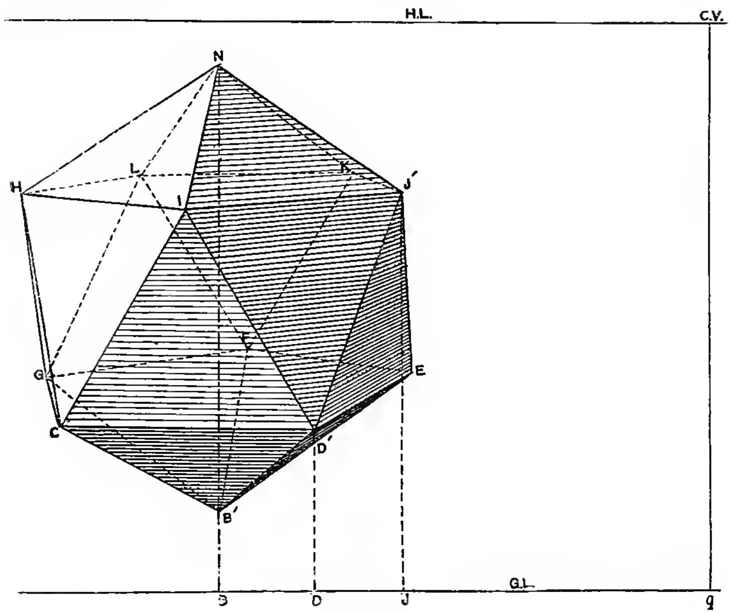


Fig. 136

of the G.P., and set up 3′ above it for the height of the H.L. Set off $b'q' = bp$, and draw a vertical line at q' to find C.V. We know that C.V. is the elevation of the station point, and therefore that lines drawn from the various corners of the figure to C.V. must give the visual rays in elevation.

Set off $q'J' = qJ$, and draw a vertical line at J′ to meet the line drawn from j' to C.V. in j''. Then j'' is the position of the point j on the P.P., *i.e.* where the visual ray from the point j to spectator's eye intersects the P.P. [9].

Now find the G.L., H.L., and C.V (Fig. 136), to a scale of $1'' = 1'$. We shall not require the P.D.

Set off qJ = twice $q'J'$ (Fig. 135). Draw a vertical line at J, and set off JJ' = twice $J'j''$ (Fig. 135). Then J' is one of the points of the solid. Determine the other points in a similar manner, and join them, as shown.

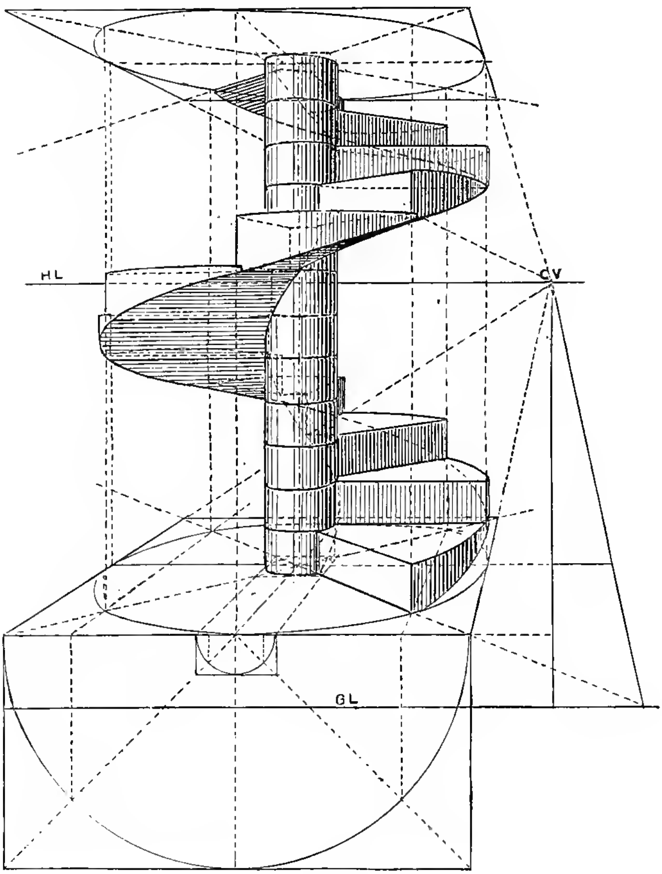


Fig. 137

Students in the advanced stage will also be required to work out problems of a similar nature to those set in the elementary

stage, but more difficult in character. Fig. 137 is an illustration of Prob. 28 in an advanced form, *i.e.* its construction is based on a series of circles in parallel planes, the circles being 6′ in diameter. It represents a cylindrical staircase (generally called a spiral staircase), and consists of twelve steps, their total height being 8′. The axis is 5′ from P.P., and 4′ to the spectator's left.

Distance of P.P. = 10′. *Height of eye* = $4\frac{1}{2}$′. *Scale* $\frac{1}{2}$″ = 1′.

It is constructed as follows:—First, find the circle in perspective on the G.P., and at a height of 8′ above it, construct a similar circle parallel to it. The construction for finding the eight points in these circles is shown. Join the corresponding points in each circle by vertical lines, and divide each line, as well as the axis, into twelve equal parts. Draw parallel circles through each of these divisions, as shown in Prob. 28. The upper part of each step occupies one-eighth of each circle; the lines forming the front of each step radiate from the corresponding divisions on the axis.

Lines of Double Curvature.—A curved line that cannot be contained by a plane is said to be a *line of double curvature;* such lines are illustrated in Fig. 137, viz. one on the surface of the larger cylinder, and the other on the surface of the central shaft. The particular curves in this figure are called helices, and theoretically are traced by the outer end of a line of fixed length drawn at right angles to the axis; this line rotates at a uniform rate round the axis, while its other end moves at a uniform rate along the axis. A similar curve could be drawn by joining the outer corners of the steps by a fair curve.

The outline of a Sphere can only be accurately represented in perspective by a circle, when its centre is directly opposite the eye, as will be seen from the following illustration.

Let A (Fig. 138) represent the plan of a sphere, P.P. the position of the picture plane, and S the station point. Draw from S tangents to the circle, forming the plan of the sphere, as *b*S, and *d*S, and join *bd*. Then S*bd* will represent the plan of a right cone tangential to the sphere, and the P.P. will form a

section through this cone at the line *ef*. Now the only section of a right cone that will give a circle is one at right angles to its axis, and the P.P. could make such a section only if the centre of the sphere were opposite the eye. In every other position the perspective representation of the outline of a sphere would be an ellipse.

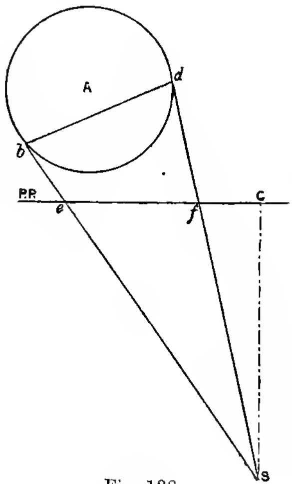


Fig. 138

EXERCISES

Practice

The scale to be used in working all these problems is half an inch to one foot.

The eye is to be, in each case, 12′ *distant from the* P.P., *and* 4′ *above the* G.P.

1. Draw a square 5′ × 5′ in an ascending plane, inclined to the G.P. at an angle of 40°; its nearest edge to rest on the G.P., parallel to the P.P., and 4′ from it; its nearest corner to be 2′ to the spectator's left.

Through the centre of the square, and at right angles to it, draw a line 4′ long, projecting to a distance of 2′ on each side of the square.

2. Draw a regular octagon of 2′ side in a plane, at right angles to the P.P., and inclined at an angle of 30° with the G.P. on the right; its lowest edge to rest on the G.P., and the nearer end of this edge to be 4′ from the P.P., and 2′ to the spectator's right.

Through the centre of the octagon, and at right angles to it, draw a line 4′ long, its lower end to be on the G.P.

3. Draw a regular hexagon of 2′ side in a descending plane, inclined to the G.P. at an angle of 36°; its farthest edge to rest on the G.P., and the left-hand end of this edge to be 2′ to the spectator's right, and 4′ within the picture.

Through the centre of this hexagon, and at right angles to it, draw a line 3′ long, projecting to an equal distance on each side.

4. Draw a square 5′ × 5′, contained by an oblique plane, the H.T. of which is inclined to the P.P. at an angle of 40° on the left; the angle of inclination of the oblique plane to the G.P. to be 36° towards the spectator's right; the lowest corner of the square to rest on the G.P., 3′ from the P.P., and 2′ to the spec-

tator's left; the nearer lower edge of the square to be inclined to the H.T. of the oblique plane at an angle of 35°.

Through the centre of the square, and at right angles to it, draw a line 5′ long, projecting to an equal distance on each side of the square.

5. Draw the object shown in Fig. 123, in a similar position to that shown in Fig. 125.

6. Draw the object shown in Fig. 127, in a similar position to that shown in Fig. 128.

Theory

1. In making a perspective drawing, how would the relative positions of the eye, the picture plane, and the object, influence the scale of your drawing [50]?

2. Upon what optical fact is the science of perspective entirely dependent [7]?

3. When can a sphere be accurately represented in perspective by a circle? What would be the form of the representation in other cases, and why (Fig. 138)?

4. How is the centre of a vanishing line determined [88]?

5. Upon what principle are vanishing lines determined [73]?

6. What is an oblique line [63]? How would you determine the vanishing point for an oblique line [68]?

7. What planes have no vanishing lines [55]?

8. Name, according to their characteristics, the various planes that occur in perspective [55-62].

9. Describe by a sketch any method of finding the vanishing line of an oblique plane [73, 74].

CHAPTER XI

SCIOGRAPHY OR THE PROJECTION OF SHADOWS

Introduction

Note.—The articles in this chapter are numbered in continuation of those in Chapter IX.

The numerals in brackets refer to the articles numbered in Chapters II. and IX., and in this chapter.

Sciography (from *skia*, a shadow, and *grapho*, I draw) is the art of drawing shadows correctly.

89. *A shadow* upon any object is caused by another object intercepting light which would otherwise fall upon it; it is consequently the projection of the outline shape of the object which intercepts the light upon the object that is deprived of that portion of the light.

90. *Shade.*—The parts of the object which receive no direct illumination are said to be in shade.

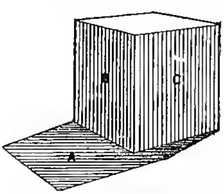


Fig. 139

Fig. 139 is the drawing of a cube with its shadow cast upon a horizontal plane. A is the *shadow*, and the faces B and C are in *shade*, as they receive no direct light.

91. Shadows may be conveniently divided into two kinds, viz. those projected from a source of light at a great distance, as the sun, and those projected from a source of light comparatively near—a lighted candle for

instance. In any case the source of light is assumed to be a point.

92. When shadows are cast by the sun, the rays of light are practically parallel to each other.

93. When shadows are cast by a lighted candle, the rays of light diverge from the source of light.

We will first take into consideration those shadows that are cast by parallel rays of light.

SHADOWS CAST BY THE SUN'S RAYS

The sun may occupy, with regard to our picture, one of the three following positions:—

It may be (1) *in the plane of the picture;* in which case the shadow of a vertical line will be parallel to the picture. (2) *Behind the picture;* in which case the shadow of a vertical line will be cast towards the spectator. (3) *In front of the picture;* in which case the shadow of a vertical line will be cast away from the spectator.

94. Before we can project the shadow of an object, we must know the relative positions of the following—the spectator, the picture, the object, the source of light, and the surface which receives the shadow.

95. THE PLANE OF PROJECTION.—When the surface which receives the shadow is a plane, it is called the *plane of projection, e.g.* the plane containing the shadow A (Fig. 139) is a plane of projection, the shadow being *projected* upon it.

96. VANISHING POINT OF THE SUN'S RAYS.—When the sun is behind the picture, the rays of light, being receding parallel lines, must be represented by lines converging to a vanishing point.

We know that a vanishing point represents on the picture plane a point at an infinite distance [13], and it is clear that in this case the vanishing point represents in the picture the position of the sun's centre, and is called *the vanishing point of the sun's rays.*

97. When the sun is behind the spectator, rays of light passing into the picture, being parallel, will again be represented by lines converging to a vanishing point; the position of this point will be determined by the intersection with the picture plane of the ray of light passing from the sun through the station point, *i.e.* of a vanishing parallel of the sun's rays [13].

98. When the sun is in the plane of the picture, the rays of light will be parallel to the picture plane, and will therefore be drawn geometrically parallel to each other [21].

99. THE SEAT OF THE LUMINARY.—A line drawn from the

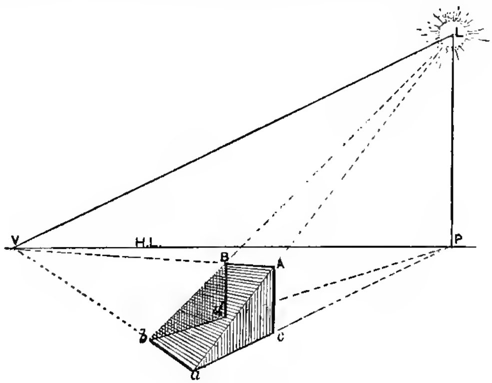


Fig. 140

centre of the luminary, at right angles to the plane of projection, will meet it in a point called *the seat of the luminary; e.g.* let us assume the ground plane (Fig. 140) to be the plane of projection, and L the vanishing point of the sun's rays. As a vanishing point represents a point at an infinite distance [13], and the H.L. the apparent termination of the ground plane in infinity. Draw a vertical line from L to meet H.L. in P. Then P is *the seat of the luminary* L.

100. THE PLANE OF SHADE.—Let A*c* and B*d* (Fig. 140) represent two vertical lines. Draw lines from P through *c* and *d* to meet lines drawn from L through A and B in *a* and *b*. Then *ca* is the shadow on the ground plane of the line *c*A, and *db* of the line *d*B. Now if a shadow is cast by a right line,

there must be a portion of the plane containing the line, its shadow, and the luminous point, which receives no direct light; *e.g.* the planes B*bd* and A*ac*, hence these planes are called *planes of shade*, and if produced would both pass through the luminous point L.

101. VANISHING LINE OF THE PLANE OF SHADE.—As A*ac* and B*bd* (Fig. 140) represent two parallel planes of shade, they must evidently converge towards the vanishing line LP [33], which is consequently called the *vanishing line of the plane of shade.*

Let the points A and B (Fig. 140) be joined by a horizontal line; the shadow of this line is found by joining *ab;* and we see that AB*ba* is also a plane of shade.

102. *To find the vanishing line for a plane of shade, join the vanishing point of the sun's rays to the vanishing point of the line that casts the shadow.*

As AB (Fig. 140) is a receding horizontal line, its V.P. must be on the H.L. [20]. Produce AB to meet the H.L. in its V.P. (V), and join VL. This is the vanishing line required.

103. The student should notice that the shadow of a line is the intersection of the plane of shade with the plane of projection; *e.g.* in (Fig. 140) the G.P. is the plane of projection, and *ac* (the shadow of the line A*c*) is the intersection of the plane of shade with the plane of projection. The line *ab* is another instance.

104. THE SEAT OF A POINT.—The point where a line drawn from a point, at right angles to a plane, meets the plane, is called the *seat of the point* upon that plane. It is in fact the *orthogonal projection* of a point upon a plane.

Assume the line AB (Fig. 140) to be suspended in mid air. Before we could determine its shadow, we should have to determine the seat of each extremity (A and B) on the plane of projection; *e.g.*, *c* is the seat of A, and *d* is the seat of B.

We will take another illustration. Let the vertical plane *D* (Fig. 141) be the plane of projection, and let A*b* be a line at

right angles to it. The sun is supposed to be in front of the picture, *i.e.* behind the spectator, and a ray of light passing from the centre of the sun through the station point is supposed to meet the picture in L (the V.P. of the sun's rays). A vertical line drawn from L will meet the H.L. in P (the seat of L), therefore PL is the V.L. for all vertical planes of shade [100].

We will first find the shadow of the line A*b* by means of this V.L.

Find the seat of the point A on the G.P. as follows:—Draw

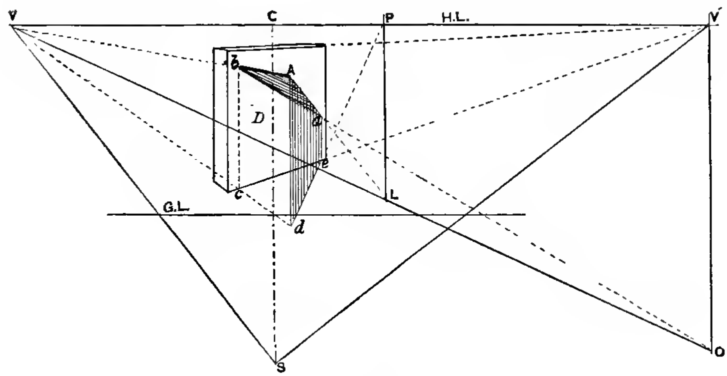


Fig. 141

a vertical line from *b* to meet the G.P. in *c*. As A*b* is a receding horizontal line, produce it to H.L. to find its vanishing point (V). Draw a line from V through *c* to meet a vertical line drawn from A in *d*. This is the projection (or seat) of A on the G.P. Draw a line from *d* to P to meet the lower edge of the plane *D* in *e*. Draw a vertical line at *e* to meet a line drawn from A to L in *a*. Join *ba*, which is the shadow of the line *b*A.

We will now find the shadow of the same line by means of the oblique plane of shade.

The plane *D* being given, we assume its V.P. (V′) to be known. At V′ draw a vertical line for the V.L. of the plane *D*. Draw a line from V through L to meet the V.L. of *D* in O. Then VO is the V.L. for the oblique plane of shade [102]. Draw a line from *b* to O to meet a line from A to L in *a*. Then

ba is the shadow, *i.e.* the intersection of the plane of shade with the plane of projection [103].

105. All shadows are found by projecting on the plane of projection certain points on the outline of the object which intercept the light; then by joining these points we obtain the outline of the shadow. Now it is evident, from the illustration we have just taken, that the point *a* which determines the shadow of the line A*b*, can be found by projecting A on the plane *D* by means of either a vertical or an oblique plane of shade, but as the construction for a vertical plane of shade is more simple and occupies less space than that required for an oblique plane of shade, we will use it for the present.

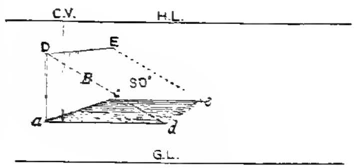


Fig. 142

Let *B* (Fig. 142) represent a vertical plane inclined to the P.P., and let the G.P. be the plane of projection; also let the sun be in the plane of the picture [98] at an altitude of 30° above the G.P., on the spectator's left.

Draw a line from D, at an angle of 30° with the G.P., to meet a horizontal line from *a* in *d*. Find the point *e* in a similar manner, and join *de*. Then *aed* is the shadow of the plane *B*.

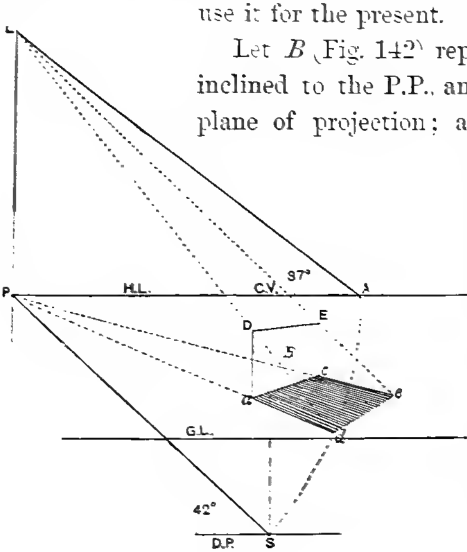


Fig. 143

Let *B* (Fig. 143) represent a similar plane, and the G.P. the plane of projection as before: but let the sun be behind the picture [96], in a vertical plane inclined to the P.P. at an angle

of 42° to spectator's left, and at an altitude of 37° above the G.P.

Draw a line making an angle of 42° at the station point (S) with D.P., and produce it to meet H.L. in P. Draw a vertical line at P for the V.L. of the plane of shade. With P as centre, and radius PS, draw an arc to meet H.L. in A. Then PA is the distance of the spectator from the V.L. Set off at A, with H.L., an angle of 37°, and produce it to meet the V.L. in L. Then L. is the V.P. of the sun's rays.

Draw a line from P through *a* to meet a line drawn from L through D in *d*. Find the point *e* in a similar manner, and join *de*. Then *aced* is the shadow of the plane *B*.

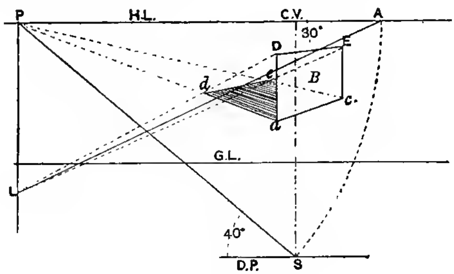


Fig. 144

Let *B* (Fig. 144) represent a similar plane, and let the G.P. still be the plane of projection; but let the sun be in front of the picture [97] in a vertical plane, receding to the left and inclined to the P.P. at an angle of 40°, and at an altitude of 30° above the G.P.

Draw a line making an angle of 40° at S with D.P., and produce it to meet H.L. in P. Draw a vertical line at P for the V.L. of the plane of shade. With P as centre, and radius PS, draw an arc to meet H.L. in A. Set off at A with H.L. an angle of 30°, and produce it to meet the V.L. in L. Then L is the V.P. of the sun's rays.

Draw a line from *a* to P to meet a line drawn from D to L in *d*. Find the point *e* in a similar manner, and join *de*. Then *adec* is the shadow of the plane *B*.

106. *If a line is parallel to the plane of projection the line and its shadow are also parallel to each other, e.g.* de *is perspectively parallel to* DE *in* Figs. 142, 143, and 144.

Note.—The numbered articles are continued at p. 228.

Problem 69

Project the shadows of the two rectangular solids A and B (Fig. 145); *the sun to be in the plane of the picture, and at an altitude of* 60° *above the* G.P. *on the spectator's right.*

Find the seat of the point D in the following manner :—Draw

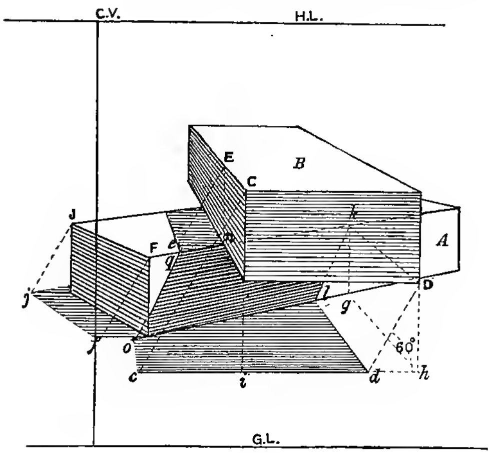


Fig. 145

a line from D towards C.V., meeting the upper edge of the block *A* in *k*. Draw a vertical line from *k* to meet the lower edge of *A* in *g*. Draw a line from C.V. through *g* to meet a vertical line from D in *h*. Then *h* is the seat of D.

Draw a line from D, at an angle of 60° with the G.P., to meet a horizontal line from *h* in *d*. Then *d* is shadow of the point D. Determine in a similar manner the points *c, f, j*, the shadows of the points C, F, J. Join *cd* and *fj*.

Draw a line from *d* to C.V., meeting the lower edge of *A* in *l*. Join *kl*. Draw a line from *e* to C.V., meeting a horizontal line from *f*. Draw a line from *j* towards the right hand V.P. of the block *A*.

The upper surface of the block *A* is the plane of projection for a portion of the shadow of block *B*. Find *n*, the seat of E on this plane, and draw a line from E parallel to D*d* to meet a horizontal line from *n* in *e*. Draw a line through *e* from C.V., meeting the upper edge of *A* in *q*. Produce the lower edge of *A* to meet the receding line from *e* in *o*. Join *qo*.

In the four following illustrations a similar model is given, viz. three rods joined together at right angles so as to form a double cross. This model gives us three lines in different positions with reference to the P.P. We will project its shadow on planes in different positions.

Problem 70

1. *Project upon the* G.P. *the shadow of the double cross A* (Fig. 146); *the sun to be in the plane of the picture, and at an altitude of* 50° *above the* G.P. *on spectator's left,* i.e. *in the direction of the arrow* R.
2. *Also of a similar cross B upon the plane C.*

1. Determine the seat of the point D in *e*. Draw a line

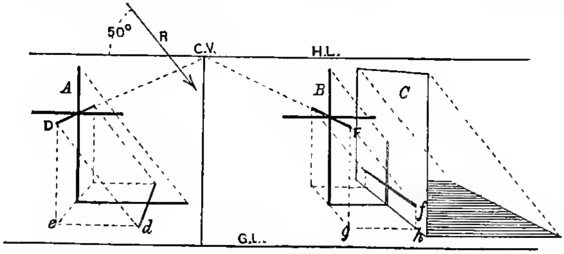


Fig. 146

from D parallel to R to meet a horizontal line from *e* in *d*.

Determine the other points in a similar manner, and join them, as shown.

2. Determine the seat of the point F in *g*. Draw a horizontal line from *g* to meet the lower edge of the plane *C* in *h*. Draw a line from F parallel to R to meet a vertical line on *h* in *f*. Determine the other points in a similar manner, and join them, as shown.

Problem 71

1. *Project the shadow of the double cross A* (Fig. 147) *upon the inclined plane B; the sun to be in the plane of the picture, and the light to come in the direction of the arrow* R.
2. *Also of a similar cross C upon the inclined plane D.*

1. Determine the seat of the point E in *f*. Draw a horizontal

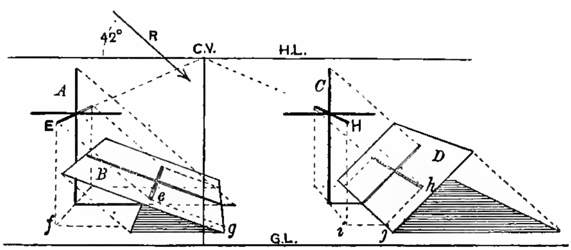


Fig. 147

line from *f* to meet the lower edge of the plane *B* in *g*. Draw a line from E parallel to R to meet a line drawn from *g* parallel to the nearer edge of *B* in *e*. Determine the other points in a similar manner, and join them, as shown.

2. Determine the seat of the point H in *i*. Draw a horizontal line from *i* to meet the lower edge of the inclined plane *D* in *j*. Draw a line from H parallel to R to meet a line drawn from *j* parallel to the nearer edge of *D* in *h*. Determine the other points in a similar manner, and join them, as shown.

Problem 72

1. *Project the shadow of the double cross B* (Fig. 148) *on the* G.P.; *the sun to be in front of the picture in a vertical plane receding to the left and inclined to the* P.P. *at an angle of* 50°, *and at an altitude of* 30° *above the* G.P.
2. *Also of a similar cross C upon a vertical plane D, and of D on the* G.P.

Draw a line making an angle of 50° at the station point (S)

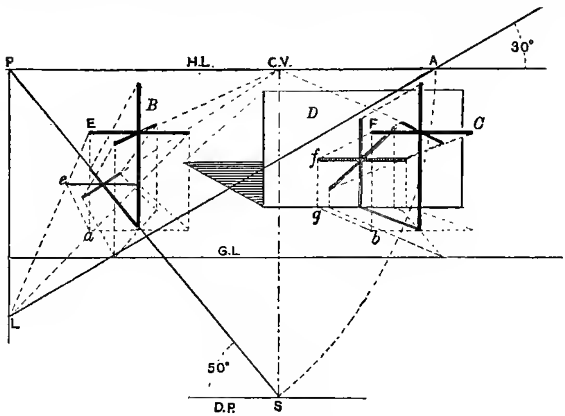


Fig. 148

with D.P., and produce it to meet H.L. in P. With P as centre, and its distance from S as radius, draw an arc to meet H.L. in A. Set off an angle of 30° at A with H.L., and produce it to meet a vertical line from P in L. Then L is the V.P. of the sun's rays, and LP the V.L. of the plane of shade.

Find *a*, the seat of E. Draw a line from *a* to P to meet a line drawn from E to L in *e*. Find the other points in a similar manner, and join them, as shown.

2. Find *b*, the seat of F. Draw a line from *b* to P to meet the lower edge of the plane *D* in *g*. Draw a vertical line from *g* to meet a line drawn from F to L in *f*. Find the other points in a similar manner, and join them, as shown.

PROBLEM 73

1. *Project the shadow of the double cross B* (Fig. 149) *upon the oblique plane C; the sun to be in front of the picture, in a vertical plane receding to the right and inclined to the* P.P. *at an angle of* 60°, *and at an altitude of* 35° *above the* G.P.
2. *Also of a similar cross D upon a vertical plane E.*

1. Draw a line making an angle of 60° at S with D.P., and

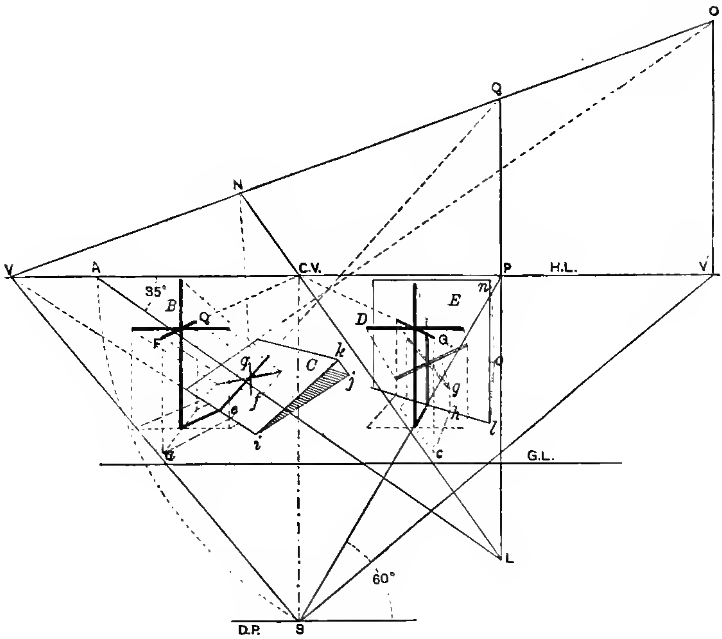


Fig. 149

produce it to meet H.L. in P. With P as centre, and its distance from S as radius, draw an arc to meet H.L. in A. Draw a line making an angle of 35° at A with H.L., and produce it to meet a vertical line from P in L. Then L is the V.P. of the sun's rays, and LP the V.L. of the plane of shade.

Produce the lower edge of the plane *C* to meet H.L. in V, the V.P. for its horizontal edges. Also find V′, the V.P. for

horizontal lines at right angles to its horizontal edges. Determine O, the A.V.P. for the oblique edges of the plane *C* [77]. Join VO. This is the V.L. of the plane *C*.

Produce LP to meet VO in Q. Then Q is the V.P. for lines common to the plane *C* and the vertical planes of shade [103].

Find *a*, the seat of F. Draw a line from *a* to P, meeting the lower edge of plane *C* in *e*. Draw a line from *e* to Q to meet a line drawn from F to L in *f*. Determine the other points in a similar manner, and join them, as shown.

Let us test the shadow of the arm FQ by means of the V.L. for oblique planes of shade. Draw a line from L through C.V. to meet VO in N. Then LN is the V.L. for the oblique plane of shade containing FQ [102], and N is the V.P. for the line common to plane *C* and this oblique plane of shade [103], consequently *fq* produced should vanish at N.

To find the shadow of the plane *C*, draw a line from *i* to P to meet a line drawn from *k* to L in *j*. Draw a line from *j* towards V.

2. Find *c*, the seat of G. Draw a line from *c* to P to meet the lower edge of the plane *E* in *h*. Draw a vertical line from *h* to meet a line drawn from G to L in *g*. Determine the other points in a similar manner, and join them, as shown.

The shadow of the plane *E* is found in a similar manner to that described for plane *C*.

Problem 74

Project the shadows of the two rectangular solids A and B (Fig. 150); *the sun to be behind the picture, in a vertical plane inclined to the* P.P. *at an angle of* 30° *to the spectator's left, and at an altitude of* 40° *above the* G.P.

The working points, etc., for drawing these blocks and shadows were set out to a scale of $\frac{1}{2}'' = 1'$, but as they cannot be shown in the space at our disposal, they are set out in a supplementary diagram drawn to a scale of $\frac{1}{16}'' = 1'$.

In this supplementary diagram s is the station point, v and v' are the V.P.'s for the block A, and v, $a.v.p.$, and $a.v.p.'$ are the V.P.'s for the block B.

Draw a line making an angle of 30° with $d.p.$ at s, and

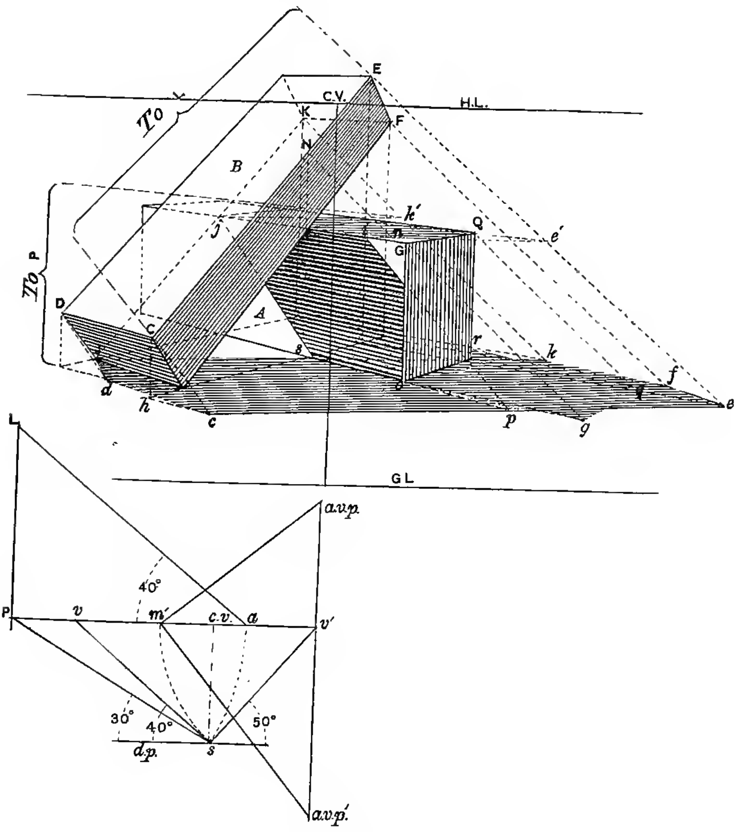


Fig. 150

produce it to meet $h.l.$ in P. With P as centre, and its distance from s as radius, draw an arc to meet $h.l.$ in a. Draw a line, making, at a, an angle of 40° with $h.l.$, and produce it to meet a vertical line drawn at P in L. Then L is the V.P. of the sun's rays, and PL the V.L. of the plane of shade.

Set off the distances between these points to eight times this scale in the larger drawing, commencing at C.V.

Find *h*, the seat of the point C. Draw a line from P through *h* to meet a line drawn from L through C in *c*. Find *d*, *k*, *e*, *f*, *q*, *g*, the shadows of the points D, K, E, F, Q, G, in a similar manner, and join them, as shown. The shadow of the corner at *g* is found by joining *g* with *o* and *q*. Join *d*J and J*k*, cutting the lower edge of *A* in *s*. Join *js*. Draw a line from *q* to the left-hand V.P. of block *A* to find *r*.

Now let the upper surface of block *A* be the plane of projection, and on it project the shadows of the points K, E, N, in the points *k'*, *e'*, *n*. Join *k'j*, also *e'n*, and produce the line to meet the edge of block *A* in *i*. Produce the line *so* to meet *ce* in *p*, and join *ip*.

Problem 75

Project the shadows of the two solids A and B (Fig. 151); *the sun to be behind the picture in a vertical plane inclined to the* P.P. *at an angle of* 50° *to the spectator's left, and at an altitude of* 48° *above the* G.P.

This group of blocks is drawn to a scale of $\frac{1}{2}'' = 1'$. The supplementary diagram of the working points, etc., is drawn to a scale of $\frac{1}{16}'' = 1'$.

The points *v* and *v'*, in the supplementary diagram, are the V.P.'s for the horizontal lines of the hexagonal prism (*A*); *a.v.p.* is the A.V.P. for the axis of the hexagonal pyramid (*B*), and the line joining *v* and *a.v.p.'* is the V.L. for its base.

Draw a line making an angle of 50° at *s* with *d.p.*, and produce it to meet *h.l.* in P. With P as centre, and its distance from *s* as radius, draw an arc to meet *h.l.* in *a*. Draw a line making an angle of 48° at *a* with *h.l.*, and produce it to meet a vertical line at P in L. Then L is the V.P. of the sun's rays, and PL the V.L. of the plane of shade.

Set off, in the larger drawing, the distances between these points to eight times this scale, commencing at C.V.

Find *n*, the seat of the point C. Draw a line from P through *n* to meet a line from L through C in *c*. Project the shadows of the other points in a similar manner. The points are marked

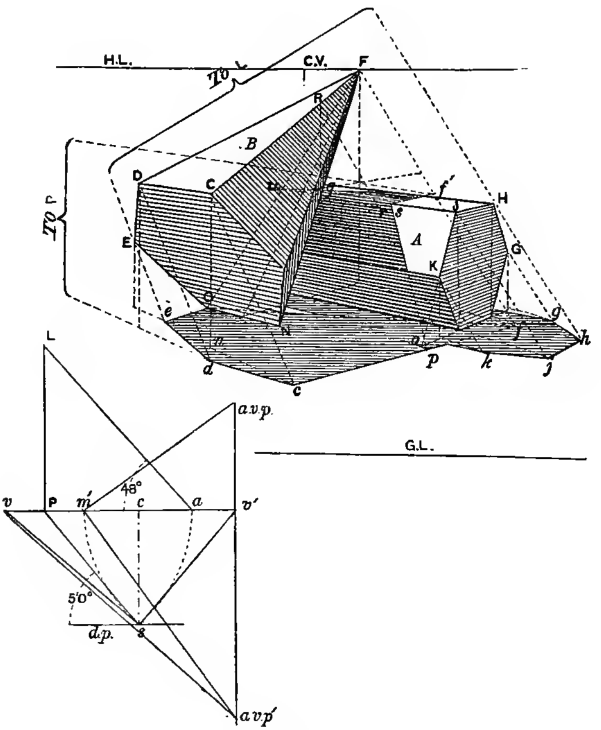


Fig. 151

with capitals, and their shadows with corresponding letters in small italics; *e.g.* the shadow of the vertex F on the G.P. is *f*, and *f′* is its shadow on a plane containing the upper surface of the solid *A*. Join O and *c* to *f*. Draw lines from *k* and *g* to the left-hand V.P. of the solid *A*.

Produce JK to meet the lowest right-hand edge of the prism produced in *o*. Draw a line from the left-hand V.P. through *o*

to meet the line *cf* in *p*. The edge NF of the pyramid intersects the upper edge of the prism in *q*. Draw a vertical line from this point to meet the edge CF in R. Find the shadow of R on a plane containing the upper surface of the prism in *r*. Join *rf′*, intersecting the upper edge of the prism in *s*. Join *sp* and *uf′*.

Problem 76

Project the shadows of the two solids A and B (Fig. 152); *the sun to be in front of the picture in a vertical plane receding to the right, and inclined to the* P.P. *at an angle of* 35°, *and at an altitude of* 30° *above the* G.P.

This group of blocks is drawn to a scale of $\frac{1}{3}'' = 1'$. The supplementary diagram of the working points, etc., is drawn to a scale of $\frac{1}{18}'' = 1'$.

The point *v*, in the supplementary diagram, is the V.P. for the H.T. of the plane containing the front of the solid *B*, and *v′* the V.P. for horizontal lines at right angles to the H.T. The line joining *v* with *a.v.p.* is the V.L. for the plane containing the front of *B*, and *a.v.p.′* the A.V.P. for lines at right angles to this plane.

Draw a line making an angle of 35° with *d.p.* at *s*, and produce it to meet *h.l.* in P. With P as centre, and P*s* as radius, draw an arc to meet *h.l.* in *a*. Draw a line making an angle of 30° at *a* with *h.l.*, and produce it to meet a vertical line at P in L. Then L is the V.P. of the sun's rays, and LP the V.L. of the plane of shade.

Set off the distances between these points to six times this scale in the larger drawing, commencing at C.V.

Assume any point F in the curve forming the top of the cylinder *A*, and find its seat in *n*. Draw a line from *n* to P, meeting a line from F to L in *f*. Select any number of points in the same curve, as C and E, and determine their shadows in a similar manner. Draw a fair curve through the shadows of

these points. Draw tangents to the ellipse, forming the bottom of the cylinder, towards P, to touch the curve forming the shadow of the top, as shown.

Select any number of points in the curves of the solid *B*,

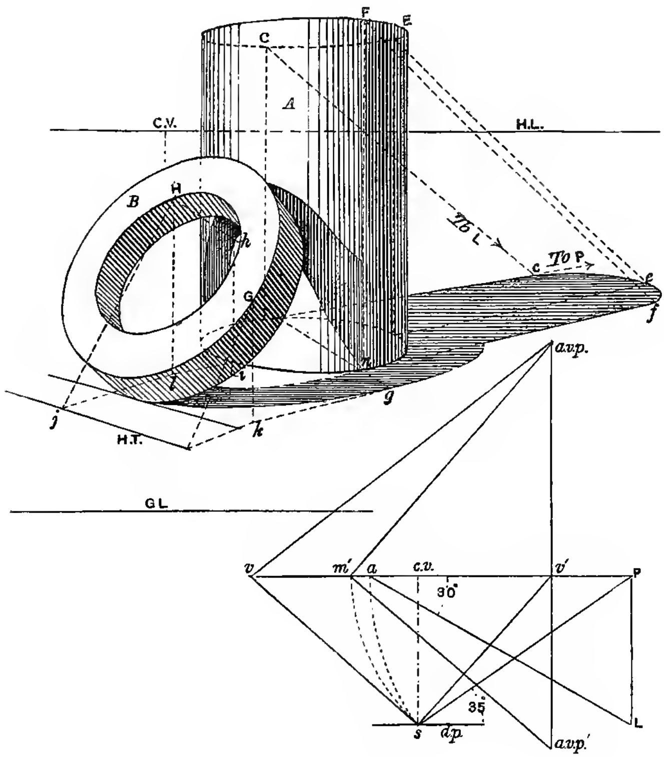


Fig. 152

e.g. let H be one of the points selected. Draw a line from A.V.P. through H to meet H.T. in *j*. Draw a line from *j* to V′ to meet a vertical line from H in *l*. Then *l* is the seat of H. Determine the seat of the point G in a similar manner in *k*. Draw a line from *l* to P to meet the base of the cylinder in *i*. Draw a vertical line at *i* to meet a line drawn from H to L in *h*. Then *h* is the shadow of H on the cylinder. Determine the

shadows of all the other selected points in a similar manner, and draw a fair curve through them for the shadow of the solid *B* on *A*.

To determine the shadow of *B* on the G.P., draw a line from *k* to P to meet a line from G to L in *g*. Select any number of points in the lower part of *B*, and determine their shadows in a similar manner. Draw a fair curve through the shadows of the points, as shown.

We will now give a few illustrations of projecting shadows by means of an oblique plane of shade.

PROBLEM 77

In Fig. 153, *B is a vertical rod, and DE a short horizontal rod fixed on the top of B, and forming the letter* T. *C is a vertical screen inclined to the* P.P. *Project the shadows of these objects by means of an oblique plane of shade; the sun to be in front of the picture in a vertical plane receding to the left, and inclined to the* P.P. *at an angle of* 45°, *and at an altitude of* 15° *above the* G.P.

V′ is the V.P. for the horizontal edges of the screen *C*, and V the V.P. for lines at right angles to it. V″ is the V.P. for the line *DE*.

Draw a line making an angle of 45° at S with D.P., and produce it to meet H.L. in P. With P as centre, and radius PS, draw an arc to meet H.L. in A. Draw a line making an angle of 15° at A with H.L., and produce it to meet a vertical line at P in L.

Determine the seat of the point *D* on the screen *C* as follows:—Draw a line from V″ through F to meet a vertical line from *D* in *c*. Draw a line from *c* to V to meet the lower edge of *C* in *h*. Draw a vertical line at *h* to meet a line from *D* to V in *a*. Then *a* is the seat of *D* on the screen *C*. Determine *b*, the seat of *E*, in a similar manner. Now V is the V.P.

for the lines *Da* and *Eb*, hence to find the V.L. for an oblique plane of shade we must draw a line from V through L to meet a vertical line from V′ in O [102]. V′O is the V.L. of the plane of projection, and VO the V.L. of the plane of shade, O being the V.P. for lines common to both planes [103].

Draw a line from *a* to O to meet a line drawn from *D* to

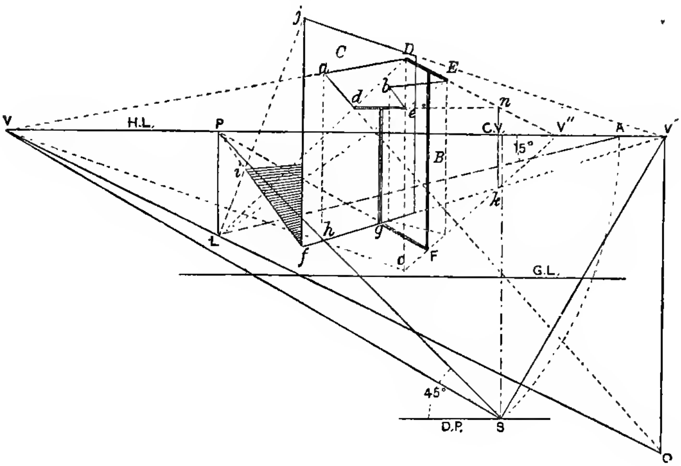


Fig. 153

L in *d*. Find *e*, the shadow of *E*, in a similar manner, and join *de*, which is the shadow of *DE*.

Draw a line from F to P, meeting the lower edge of *C* in *g*. Draw a vertical shadow of the bar *B* from *g* to meet *de*.

Draw a line from *f* to P to meet a line from *j* to L in *i*. Draw a line from *i* to V″. This gives the shadow of the screen *C* on the G.P.

If we produce the lower edge of screen *C* to meet the line from F to V″, we obtain the point *k*; a vertical line at *k* meets *DE* produced in *n*, consequently, *kn* is the line of intersection of the screen *C*, and a vertical plane containing the line *DE*, and *n* is a point common to both planes, so *DE* and *de* produced will meet in *n*.

PROBLEM 78

Fig. 154 *is the drawing of an arch to a scale of* $\frac{1}{2}'' = 1'$. *Project its shadow by means of an oblique plane of shade; the sun to be in front of the picture, in a vertical plane receding to the right, and inclined to the* P.P. *at an angle of* 70°, *and at an altitude of* 20° *above the* G.P.

In the supplementary drawing, which is drawn to a scale of

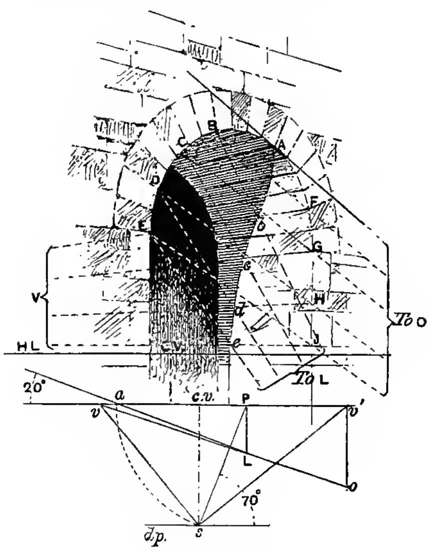


Fig. 154

$\frac{1}{16}'' = 1'$, v is the V.P. for horizontal lines receding to the left, and v' for those receding to the right.

Draw a line making at s an angle of 70° with *d.p.*, and produce it to meet *h.l.* in P. With P as centre, and Ps as radius, draw an arc to meet *h.l.* in a. Draw a line making an angle of 20° at a with *h.l.*, and produce it to meet a vertical line at P in L. Draw a line from v through L to meet a vertical line drawn from v' in o. Then $v'o$ is the V.L. of the plane containing the front of the arch, and vo is the V.L. of the plane of shade.

Set off the distances between these points to eight times this scale in the larger drawing, commencing at C.V.

Draw from O a tangent to the arch. This determines the point A from which the shadow commences. Assume any points in the arch, as B, C, D, E, and draw lines from them to O, meeting the opposite side of the opening in the points F, G, H, J. Draw lines from these points to V, meeting lines drawn from B, C, D, E to L, in the points *b*, *c*, *d*, *e*. Draw a fair curve through these points for the shadow required.

The points *b*, *c*, *d*, *e* are in planes parallel to the plane containing the front of the arch, for which V′O is the V.L.

Problem 79

Fig. 155 *is the drawing of a semicircular recess, the top of which is a quarter of a sphere, drawn to a scale of* $\frac{1}{2}'' = 1'$. *Project its shadow; the sun to be in front of the picture, in a vertical plane receding to the left, and inclined to the* P.P. *at an angle of* 76°, *and at an altitude of* 25° *above the* G.P.

In the supplementary drawing, which is drawn to a scale of $\frac{1}{16}'' = 1'$, v is the V.P. for horizontal lines receding to the left, and v' for those receding to the right.

Find the V.P. of the sun's rays, and the V.L. of the plane of shade, in a similar manner to that described in the preceding problem.

We shall require in this problem an A.M.P. corresponding to the A.V.P. (o). This is determined in the following manner:—

Draw a line through c (the C.V.) at right angles to ov', meeting it in c' (the C.V.L.). Draw a line (cs') from c at right angles to this line. With c as centre, and radius cs, draw an arc to meet cs' in s'. Join $s'c'$. With c' as centre, and radius $c's'$, draw an arc to meet the line $c'c$ produced in s''. With o as centre, and radius os'', draw an arc to meet ov' in m, which is the A.M.P. required.

Set off the distances between these points to eight times this scale in the larger drawing, commencing at C.V.

Draw from O a tangent to the arch. This determines the point A from which the shadow commences. Assume any points in the arch, as E, F, G, and draw lines from them to O, meeting the opposite side of the arch in the points C, S, J. These

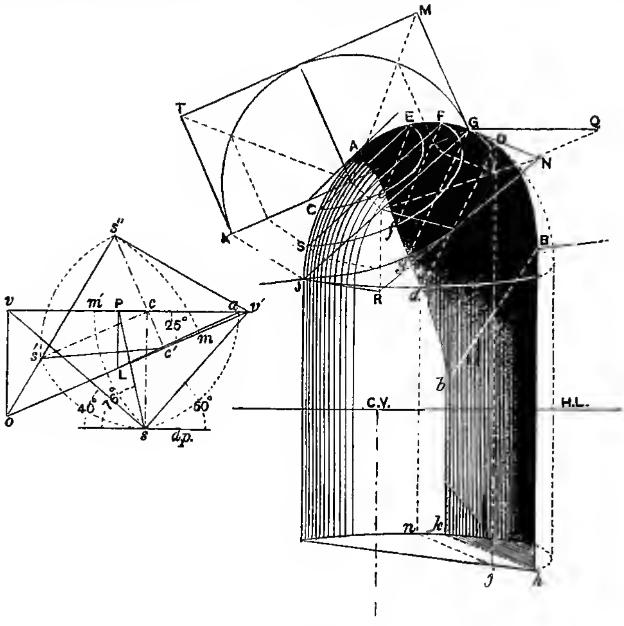


Fig. 155

lines are the front edges of planes parallel to the plane of shade, which form sections through the top of the recess. As this is a quarter of a sphere, each of these sections is a semicircle. We will draw one of these semicircles as an example.

Draw a line from G parallel to *ov'*. Draw a line from A.M.P. through J to meet this line in K. On this line describe a semicircle, and find the points, as shown (Prob. 25). Draw a horizontal line GQ = GM. Draw a line from G to V′ to meet a line from Q to M′ in N. Draw a line from N to O to meet a line from J to V′ in R. Then GNRJ is the rectangle GMTK

in perspective. Find the necessary points for drawing the semicircle, as shown (Prob. 25). Draw a semicircle in perspective on each of the other lines, as shown, in a similar manner.

Draw lines from the points E, F, G to L, meeting the semicircles in the points *e*, *f*, *g*. Draw a fair curve through these points.

To project the shadow of the remaining portion of the arch, assume any point D, and draw a vertical line D*j*. Draw a line from *j* to P to meet the back of recess in *n*. Draw a vertical line from *n* to meet a line from D to L in *d*.

The lower part of the shadow is found by drawing a line from *h* to meet the back of the recess in *k*. Draw a vertical line from *k* to meet a line from B to L in *b*. Complete the shadow, as shown.

Shadows cast by Artificial Light

We will take a similar vertical plane (*B*, Fig. 156) to that already illustrated in Figs. 142, 143, and 144, and project its shadow on the G.P. by rays of light proceeding from the lighted candle (L).

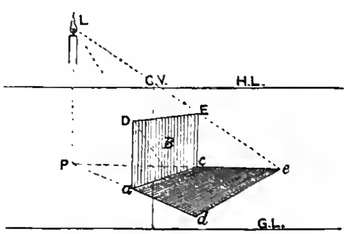


Fig. 156

Let P be the seat of L. Draw a line from P through *a* to meet a line from L through D in *d*. Then *d* is the shadow of D. Find the shadow of E in *e* in a similar manner, and join *de*. Then *aced* is the shadow of the plane *B*.

Fig. 157 represents the interior of a room, with a lighted candle (L) standing upon a table; a picture frame (F) hanging on the right-hand wall; a bell-pull (E) suspended from the ceiling; and a rod (R) leaning against the table, parallel to the wall at the farther end of the room. Project the shadows of these objects upon the different parts of the room.

First determine the seats of the corners of the table on the

floor as follows :—Draw a vertical line from A to meet a line drawn from P.D. through the lower end of the table leg in *e*. Draw a horizontal line from *e* to meet a vertical line from B in *f*. Draw lines from *e* and *f* to C.V. to meet vertical lines drawn from C and D in *g* and *h*.

Next determine the seat of L on the floor as follows :—Draw a vertical line from L to meet the top of the table in *n*. Draw

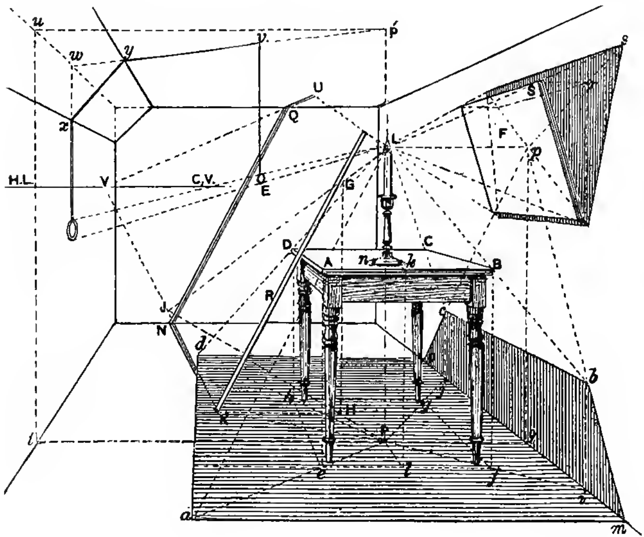


Fig. 157

a line from C.V. through *n* to meet the edge of the table in *k*. Draw a vertical line from *k* to meet *ef* in *l*. Draw a line from *l* to C.V. to meet a vertical line from L in P. Then P is the seat of L on the floor.

Draw a line from P through *e* to meet a line from L through A in *a*. Find the shadow of D in *d*, in a similar manner, and join *ad*. Draw a line from P through *f* to meet the right-hand wall in *i*. Draw a vertical line at *i* to meet a line drawn from L through B in *b*. Find the shadow of the corner C in *e* in a similar manner, and join *bc*. Draw horizontal lines from *a* and

d to meet the right-hand wall in *m* and *o*. Join *mb* and *oc* to complete the shadow of the table.

Find the seat of L on the right-hand wall as follows :— Draw a horizontal line from P to meet the wall in *q*. Draw a vertical line from *q* to meet a horizontal line from L in *p*.

Find *r*, the seat of S. Draw a line from *p* through *r* to meet a line from L through S in *s*. Find the shadows of the other corners of the frame in a similar manner, and join them, as shown.

Between the upper part of the left-hand wall and the ceiling there is an inclined plane, but the work will be simplified if we assume the wall and the ceiling to meet at right angles, as shown by dotted lines.

Find the seat of L on the ceiling in P′. Draw a line from P′ through *v* to meet the wall produced in *w*. Draw a vertical line from *w* to meet lines drawn from L through the top and bottom of the ring, as shown. Join *xy*, for the shadow on the inclined plane.

Assume any point G on the rod (R), and find its seat on the floor in H. Draw a line from P through H to meet a line from L through G in J. Join JK, cutting the end wall in N. As the rod is parallel to the end wall, so its shadow on the wall must be parallel to the rod. Therefore, draw a line from N, parallel to the rod (R), to meet the ceiling in Q. As the ceiling and floor are parallel planes, so the shadow of the rod upon them must also be parallel. Produce KJ to meet the H.L. in V. Draw a line from V through Q to meet a line from L through the end of the rod in U.

Reflection

The line M (Fig. 158) represents the edge of a mirror, called the *plane of reflection*. P is a point, and E the position of the eye. A ray of light from P, reflected from the mirror to the eye, first travels from P to *c*, and then from *c* to E. The line

bc is at right angles to the plane of reflection, and by a well-known law in physics the angle Pcb, called the *angle of incidence*, is always equal to the angle Ecb, called the *angle of reflection*.

We determine the reflection of an object by first selecting certain points contained by it, and then finding, by the above law, their apparent position on the surface of the mirror. By joining these points in a corresponding manner to those of the object, we obtain the shape of the reflected image.

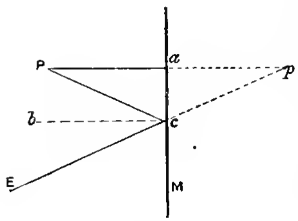


Fig. 158

The reflection of the point P will appear in the direction of the line Ep, and the distance Pa will appear equal to the distance ap. Consequently, *a line joining a point with its apparent position in a mirror is always at right angles to the plane of reflection, and bisected by it.* Hence, if we draw a line from a point to meet a mirror at right angles, and produce it in perspective to an equal distance beyond, we shall obtain the apparent reflection of the point. If we bear this in mind we shall be able to simplify the work of finding the reflection of given points, as we need not then trouble ourselves about the incident and reflected rays. This method is applicable to a mirror placed in any position whatever.

Fig. 159 represents the reflection of several objects in calm water, the surface of which is the plane of reflection. The reflections are determined by selecting certain points on the objects, and repeating their distances from the plane of reflection, below its surface, on lines at right angles to it; *e.g.* the line Aa is produced, and the distance aA' is made equal to aA; the line Bb is produced, and bB' is made equal to bB. Any other point may be determined in a similar manner.

To find the reflection of the arch. Join CD. Assume any points in the curve, E for instance. Draw a vertical line from

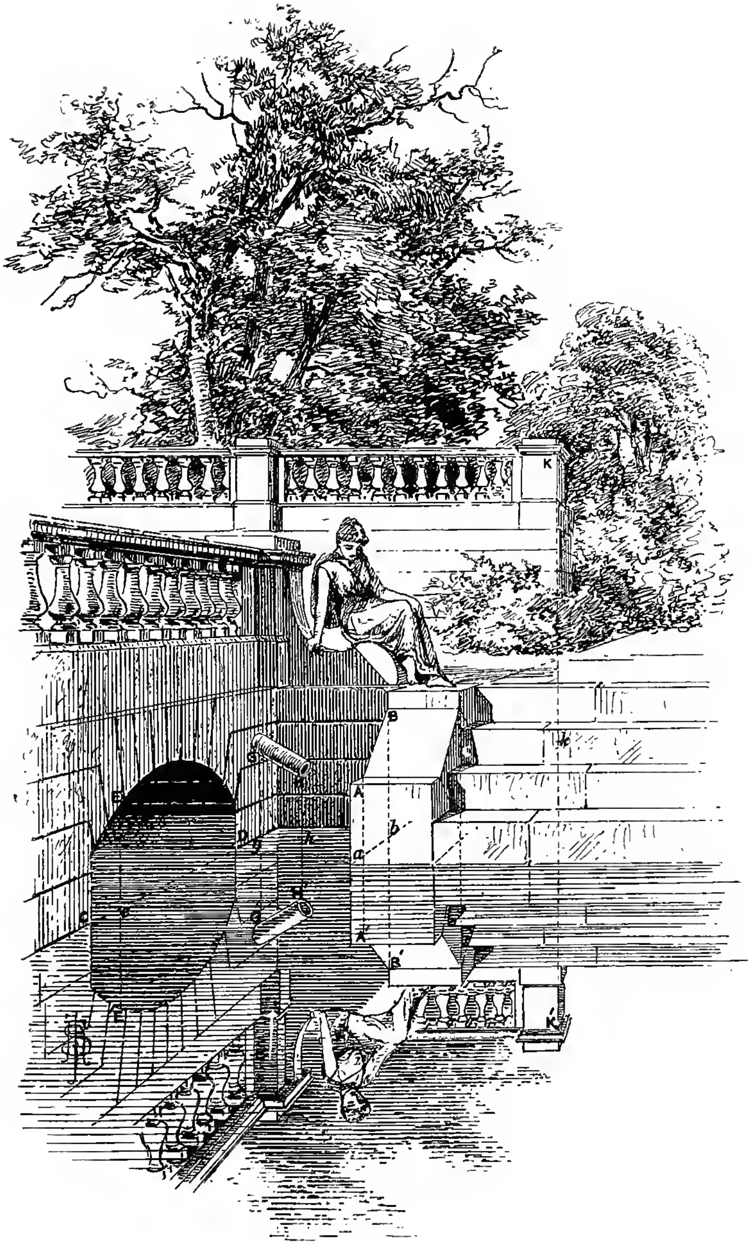


Fig. 159

E, intersecting CD in e. Make eE′ equal to eE. Determine the reflection of any other points in the curve in a similar manner, and draw a fair curve through them.

To find the reflection of the pipe. Draw a vertical line from G, intersecting CD produced in g. Draw a horizontal line from g to meet a vertical line from H in h. Make gG′ = gG, and hH′ = hH. Join H′G′, and complete the pipe, as shown.

There is a point in this drawing that is worth the student's special attention. The point K′ is the reflection of K; now the connection of K with a horizontal plane is hidden by the shrubs in front of the object to which it belongs. Without this connection we are unable to tell its distance, unless we have other data (described in Prob. 56), but we could determine its distance in this instance from its reflection; for example, we know that a point midway between K and K′ must be on the surface of the plane of reflection as k, and from this point we could determine the distance of K (Prob. 42).

Problem 80

1. *Let A* (Fig. 160) *be a triangular prism lying on the* G.P. *Assume the* G.P. *to be the plane of reflection. Show the reflection of the object.*
2. *M is a vertical plane at right angles to the* P.P. *Assume this to be the plane of reflection. Show as much of the prism and its reflection in the horizontal plane as would be reflected in this vertical plane.*

1. Draw a vertical line from k, and make $pj = kp$. Join jf and ji. Proceed in a similar manner with the opposite end of the prism, and join jl'.

2. Draw a horizontal line from f, intersecting the lower edge of the plane M in g. Make $gf' = fg$. Find the reflection of i in a similar manner, to find i'. Join $f'i'$. Draw a horizontal line from p to meet $f'i'$ in p'. Draw a vertical line at p' to meet

a horizontal line from k in k'. Join k' to i' and f' Produce the line $k'p'$ to meet a horizontal line from j and j'. Join j' to i' and f'. Find the opposite end of the prism in a similar manner and complete the drawing, as shown.

The supplementary diagram shows a plan of the prism (a), and the apparent position of its reflection (a'), as well as the position of the plane of reflection (m). Now the prism (a) forms an angle of 40° with m. Consequently, a' forms the same angle,

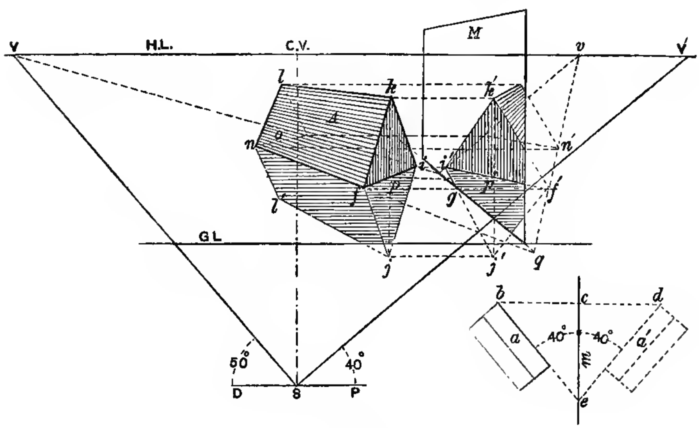


Fig. 160

as shown. So the longer edges of a' are inclined to the P.P. at an angle of 50° on the right. If we find the V.P. for lines receding at this angle, as v, we shall be able to determine the reflection of the prism by another method, which is very useful in complicated subjects ; *e.g.* produce the line nf to meet the lower edge of M produced in q. Draw a line from q to v (the V.P. of the reflection in the vertical mirror) to meet horizontal lines drawn from f and n in f' and n'. The reason for this is seen in the supplementary drawing, where bd is the base of an isosceles triangle bisected by the plane of the mirror, while be and de are the two equal sides of the triangle. Consequently, $dc = bc$.

Problem 81

Let A (Fig. 161) *be an equilateral triangular prism with the edge* BC *receding to the right at an angle of* 55° *with the* P.P., *and let M be a vertical plane of reflection, parallel to the picture. Determine the reflection of the prism.*

If we refer to the supplementary diagram, in which *a* is a plan of the prism, *a'* the apparent position of its reflection, and *m* the position of the plane of reflection, we shall see that, in

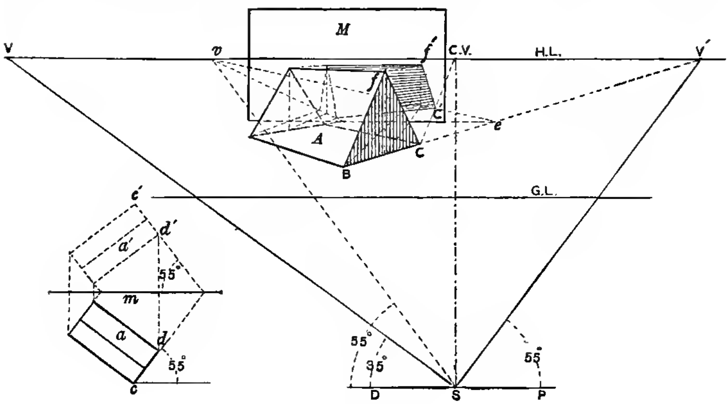


Fig. 161

accordance with the law of reflection already laid down, the reflection *d'c'* of the edge *cd* will form an angle with the P.P. of 55° to the left, and that lines joining the points of the prism with the corresponding points in the reflection are at right angles to the P.P.

Find the V.P. for horizontal lines receding to the left at an angle of 55°, as *v*. Produce the line BC to meet the lower edge of *M* produced in *e*. Draw a line from *e* to *v* to meet a line from C to C.V. in C′.

Determine all the other points, the construction for which is shown, in a similar manner, and join them to complete the reflection.

Problem 82

1. *Let A (Fig. 162) be an equilateral triangular prism, and M an inclined plane of reflection, of which the line* cd *is the picture line. Find the reflection of the prism.*
2. *Let M′ be a horizontal plane of reflection, of which the line* pq *is the picture line. Find the reflection of the prism.*

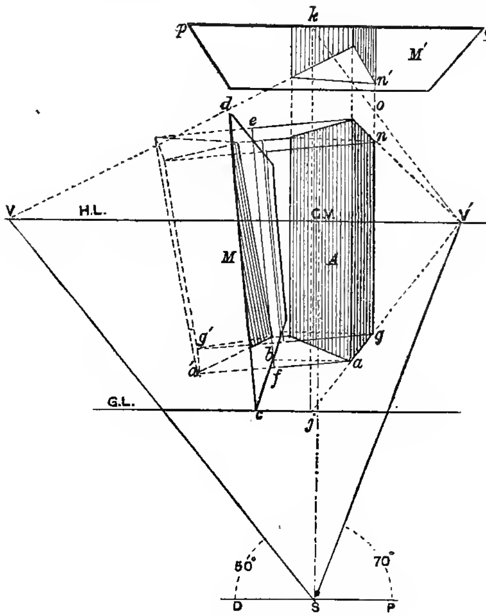


Fig. 162

1. Draw a horizontal line from *a* to meet the lower edge of *M* in *b*. Draw the line *be* parallel to *cd*. Draw a line from *a* at right angles to *be* produced, and intersecting it in *f*. Make *fa′* = *fa*. Determine the reflection of all the other points, the construction for which is shown, in a similar manner, and join them to complete the reflection.

2. Produce *ga* to meet the G.L. in *j*. Draw a vertical line from *j* to meet *pq* in *k*. Draw a line from *k* to V′. Draw a

vertical line from n, intersecting the line from k to V′ in o. Make $on' = on$. Determine the other points in a similar manner, and join them, as shown.

The following articles contain the rules for finding the V.P.'s *for the reflected image in different positions. They are numbered in continuation of those at the beginning of this chapter.*

107. The V.P. for the reflection of lines at right angles to a mirror obviously coincides with the V.P. for the original lines, for in fact they are the same lines; *e.g.* the line drawn from C to C.V. (Fig. 161) is at right angles to the mirror (M), and its reflection is a continuation of the same line.

108. The V.P. for the reflection of lines parallel to a mirror obviously coincides with the V.P. for the original lines, for they all belong to the same set of parallel lines; *e.g.* the line lk (Fig. 160) is parallel to the horizontal plane of reflection, and its reflection $l'j$, if produced, would meet H.L. in V. Also the top of the prism (Fig. 162) is parallel to the mirror (M'), and the lines of its reflection converge towards the same V.P.'s.

109. When a mirror is a vertical plane, either at right angles or parallel to the P.P., the V.P.'s for horizontal lines reflected therein is on the H.L.—for the reflections are also horizontal, and at the same distance from C.V. as the V.P.'s for the original lines, but in an opposite direction; for in both cases the original lines and their reflections are equally inclined to the P.P., but in opposite directions; *e.g.* the V.P. (v) for the horizontal lines in the reflected image (Figs. 160 and 161) will be found to be at the same distance from C.V. as the V.P. for the corresponding lines of the object, but in an opposite direction.

110. The point v (Fig. 160) is said to be the reflection of the vanishing point V. The two lines fk, nl, if produced, would meet in an A.V.P. The reflection in the H.P. of these two lines (fj, nl'), if produced, would meet in a point directly under the A.V.P., and at the same distance from the H.L. This point would be called the reflection of A.V.P.

111. To find the V.P.'s, etc., for drawing the reflection of an object, in an oblique position, in a vertical plane parallel to the P.P. In the first place, we know that the position of a reflected image in reference to the original object is always reversed. This of course is familiar to any one who has looked at himself in a mirror; his right hand appears to be his left, and *vice versa.* This being the case, we have only to reverse the relative positions of the V.P.'s, A.V.P.'s, V. lines, etc., of the original object, with reference to the C.V. and H.L., to find the corresponding points for drawing the reflection; *e.g.* the A.V.P. for the line *f*C (Fig. 161) will be on a vertical line from V′ below the H.L.; the A.V.P. for the reflection of this line (C′*f*′) will be on a vertical line at *v*, and the same distance above *v* as the A.V.P. for the original line is below V′. The same rule applies to objects, in an oblique position, reflected in a mirror in a horizontal position.

112. To find the V.P.'s, etc., for drawing the reflection of an object, in an oblique position, in a vertical mirror at right angles to the P.P. The V.P.'s, A.V.P.'s, etc., of the reflected image are transferred to corresponding positions on the opposite side of the C.V. to those occupied by the corresponding V.P.'s, etc., of the original object.

113. The following is a useful method for finding the V.P.'s, etc., for drawing the reflection of an object in an oblique position, when the mirror is in any of the following positions, viz. in a plane parallel to the P.P., in a plane at right angles to the P.P., or in a horizontal plane.

Assume the spectator to be on the opposite side of the P.P., at the required distance, and looking back towards the P.P. Then, if all the working points, etc., for drawing the original object are determined from the given data, their positions on the P.P. will represent the working points for drawing the reflection.

It should be observed that all the above rules relating to mirrors follow at once from the simple fact that every *real point* has its corresponding *reflection* opposite to itself in the mirror, and at an equal distance from the mirror on the other side.

It is generally of great assistance, as we have seen, to draw reflected points in plan *as if they were real points*, and then throw the whole into perspective.

114. When the mirror is inclined to the P.P., or to the G.P., or to both planes, we must calculate the angles that the several lines of the reflected image form with the P.P., and determine the positions of their V.P.'s, etc., as illustrated in the following problem.

Problem 83

In Fig. 163 *A is a rectangular object resting on the* G.P., *and inclined to the* P.P. *at an angle of* 50° *on the right. M is a plane of reflection inclined to the* G.P. *at an angle of* 70°, *and with its* H.T. *inclined to the* P.P. *at an angle of* 30° *on the left. Determine the reflection of the solid in the plane M.*

Scale $\frac{1}{2}'' = 1'$.

In the larger supplementary diagram, which is drawn to a scale of $\frac{1}{16}'' = 1'$, v and v' represent the *v.p.*'s of the object, and s the station point. From these points we will proceed to determine the positions of all the other working points, etc. Draw lines, making, at s with *d.p.*, an angle of 30° on the left and 60° on the right, and produce them to meet the *h.l.* in t and t'. Then t is the *v.p.* for the *h.t.* of the plane of reflection, and t' a *v.p.* for horizontal lines at right angles to it.

Draw a vertical line through t' for the *v.l.* of a vertical plane at right angles to the *h.t.* of the plane of reflection. With t' as centre, and $t's$ as radius, draw an arc to meet *h.l.* in s'. Then $t's'$ is the distance of the station point from the *v.l.* Draw a vertical line *d.p.'* through s'.

As the plane M is inclined to the G.P. at an angle of 70°, *i.e.* at 20° to the vertical, lines at right angles to M will be inclined at an angle of 70° with the vertical. Set off these angles at s' with *d.p.'*, and produce them to meet the vertical line through t' in the points p and p'. Then p is the *a.v.p.* for

lines at right angles to the plane *M*, and p' for lines parallel to *M* and at right angles to its horizontal trace.

We must now determine the *v.l.* for the reflection of the G.P., as well as for lines at right angles to it, as follows :—

The plane *M* is inclined to the G.P. at an angle of 70°,

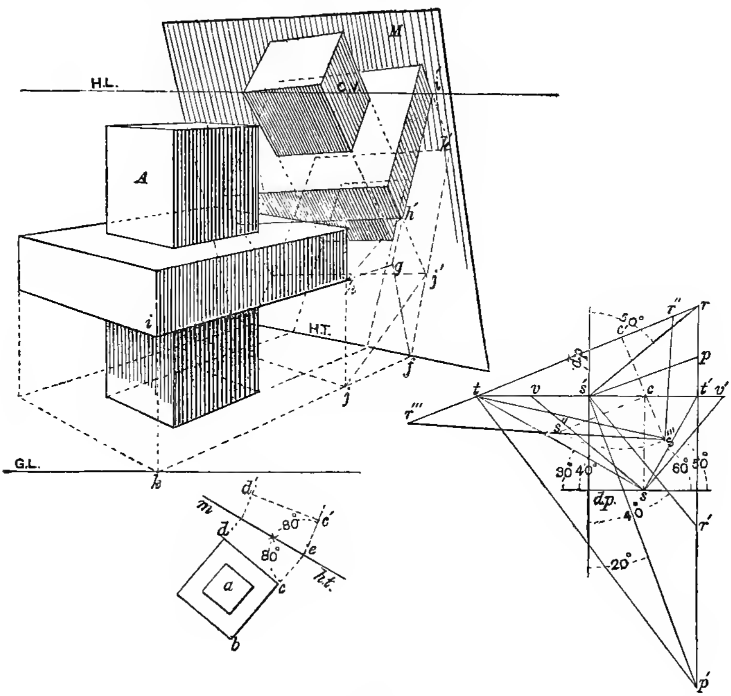


Fig. 163

therefore the reflection of the G.P. must also be inclined at an angle of 70° with *M*. Consequently the reflected G.P. will form an angle of 140° with the G.P. (70° + 70° = 140°), *i.e.* an angle of 50° with the vertical (140° − 90° = 50°), and a line at right angles to the reflected G.P. will form an angle of 40° with the vertical. Draw lines making these angles at s' with *d.p.'*, and produce them to meet the vertical line through t' in r and r'. Join rt, which is the *v.l.* for the reflected G.P., r' being the *a.v.p.* for lines at right angles to it.

We will now refer to the smaller supplementary diagram, in which a is the plan of the object, and m the position of the *h.t.* of the plane of reflection. The line ec' is the plan of the apparent position of the line ce in the reflection. Now ce is inclined to *h.t.* at an angle of 80° (50° + 30°), consequently ec' must also be inclined at an angle of 80° to it. We will now proceed to find the *a.v.p.* for the line ec', as well as for the line $c'd'$ at right angles to it, in the larger supplementary drawing.

Draw a line through c at right angles to tr, and meeting it in c'. Draw a line from c at right angles to cc', and equal in length to cs, to find s''. With c' as centre, and radius $c's''$, draw an arc to meet $c'c$ produced in s''' Join s''' to t, which is the position of the vanishing parallel for the *h.t.* of the plane of reflection, with reference to the *v.l.* for the reflected G.P. Draw a line making an angle of 80° at s''' with $s'''t$, and produce it to meet tr in r''. Also draw a line making an angle of 90° at s''' with $s'''r''$, and produce it to meet rt produced in r'''. Then r''' is the *v.p.* for the reflection of those horizontal lines that recede to the left, and r'' that of those receding to the right.

Set off the distances between these points to eight times this scale in the larger drawing, commencing at C.V. In referring to these points, we shall denote them by the capital letters corresponding to the small letters in the supplementary diagram.

Find the seats of the several points of the object A on the G.P.; *e.g.* the seats of the points i and h are k and j. Join kj, and produce the line to meet H.T. in f. Draw a line from P′ through f to meet ih produced in g. Draw lines from f and g to R, meeting lines drawn from j and h to P in j' and h'. Determine all the other points in a similar manner, and join them, as shown, to complete the reflection of the object.

The lines jf, fj' form equal angles with the plane M, and the line jj' is at right angles to the plane, consequently it is the base of an isosceles triangle.

In the Science and Art examinations (Advanced Stage)

questions are also set on the projection of shadows by means of orthographic projection.[1]

EXERCISES

Practice

Note.—The objects selected for the following exercises should be first drawn to twice the scale given.

1. Cast upon the ground plane the shadow of the prism in Fig. 62; the sun to be in the plane of the picture, and on the spectator's left at an altitude of 40° above the ground plane.

Show the reflection of the prism in the ground plane.

2. Cast upon the ground plane the shadow of the object shown in Fig. 52; the sun to be behind the picture in a vertical plane inclined to the picture at an angle of 30° on the left and at an altitude of 40° above the ground plane.

3. Project on the ground plane the shadow of the prism (Fig. 64); the sun to be in front of the picture in a vertical plane receding to the right, and inclined to the picture at an angle of 40° and at an altitude of 30° above the ground plane.

Show the reflection of the object in the ground plane.

4. Determine the reflection, in a vertical plane at right angles to the picture and 5 feet to the spectator's left, of the object shown in Fig. 55.

5. Show the reflection, in a vertical plane parallel to the picture and 10 feet from it, of the object shown in Fig. 63.

6. Cast upon the ground plane the shadow of the object shown in Fig. 125; the sun to be behind the picture in a vertical plane inclined to the picture at an angle of 30° on the left and at an altitude of 35° above the ground plane.

Show the reflection of the object in the ground plane.

7. Assume a vertical plane at right angles to the picture and 5 feet to the spectator's right in Fig. 126. Also assume a lighted candle to be placed 7 feet from the picture, 5 feet to the spectator's left, and 7 feet above the ground plane. Project the shadows of the object upon the ground plane and vertical plane.

8. Find the reflections in the ground plane of the objects shown in Fig. 152.

Theory

1. What is the difference between *shade* and *shadow* [89-90]?

[1] This subject is fully treated of in Spanton's *Complete Geometrical Course*: Macmillan and Co.

2. What is a plane of shade [100]? How is the vanishing line of a plane of shade determined [102]?

3. Explain why in the perspective representation of shadows the sun's centre may be represented by a definite point on the picture [96].

4. What is the essential difference between shadows cast by the sun's rays and those cast by a lighted candle [92-93]?

5. What is the plane of projection [95]?

6. What is the seat of a point [104]? How would the seat of the vanishing point of the sun's rays be determined, and what is it called [99]?

7. In what position would the sun be if the rays of light were geometrically parallel to each other in the picture [98]?

8. Distinguish between a *vertical* and *oblique* plane of shade [105].

9. Describe a simple method for finding the reflection of a point in a mirror placed at any angle with the picture and ground plane (p. 222).

10. How are the reflections of V.P.'s, etc., determined in a mirror placed as follows—

(*a*) In a vertical plane at right angles to the picture [112]?

(*b*) In a vertical plane parallel to the picture [111]?

(*c*) In a horizontal plane [111]?

CHAPTER XII

PERSPECTIVE APPLIED TO SKETCHING FROM NATURE

NOTE.—The numerals enclosed in brackets refer to the numbers of the articles in Chap. II.

This chapter shows how the main lines of a drawing may be set out, by the rules of perspective, directly from the object, as in sketching from nature.

We should first determine the position of the horizon line [18], which depends, in the first place, upon the height of the eye; for instance, if the spot from which we are making our sketch is situated at a high level above the ground plane [2], we should place the horizon line high upon our drawing; but if we were making our sketch from a low level—sitting in a small boat, for example—we should make the position of the horizon line correspondingly low. Again, if we were standing on the ground plane, a very convenient height to draw the horizon line would be one-third the height of the drawing for an ordinary landscape. The position of the horizon line also depends upon the amount of foreground we wish to show, as explained in Article 50.

Let us imagine we are about to sketch the view shown in Fig. 164. Having drawn a horizontal line for the horizon line (H.L.) at about one-third the height of our drawing, take a piece of string and tie a knot at each end; the distance between the knots to be equal to the length of the diagonal (*bc*) of our drawing. Place one knot between the teeth and hold the other

knot under the thumb, against the sketching block. Hold the block in a vertical position, as far from the eye as the length of the string will permit and at right angles to the string. Turn the head till the drawing or sketching block covers the subject we wish to sketch, keeping the horizon line level with the eye, *i.e.* level with the horizon. The extent of the subject covered by the sketching block will then be within the field of vision [44]. Now notice where the edges of the block appear to cut the subject. In the first place, see where the lower edge of the block appears to cut the roadway, which can be denoted by any

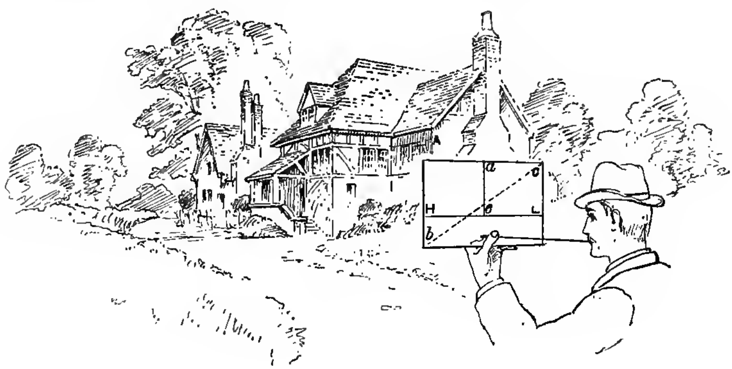


Fig. 164

marks or stones on the road. This will mark the limit of the foreground in the sketch [50]. Next notice where the vertical edges of the block appear to cut the trees in the background. These positions will mark the lateral extent of the sketch.

Slightly lower the sketching block, so that the angle A of the nearest building can be seen over the top of the block, and mark its apparent position (*a*) on the upper edge. From *a* draw a vertical line to meet the line HL in *e*. Now remove the block from its vertical position, so that we can see the whole of the subject.

We will now turn to Fig. 165, which represents the sketch to a larger scale, and in which E represents the point (*e*) which

we have just determined. Hold a pencil in a vertical position at the end of the string, and measure upon it the apparent height of EF, and transfer it to the drawing. Take a small rectangular piece of stiff paper—an ordinary visiting-card will answer the purpose—and hold it in a vertical position, with one edge covering the line EF, and one corner covering F. The card may be held at any distance from the eye; its

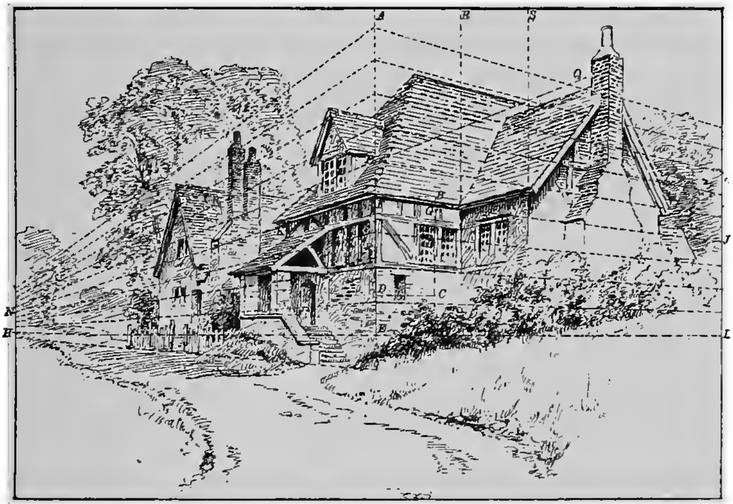


Fig. 165

position is indicated by the letters FBCD. Mark the edge of the card, where the receding line FG appears to cut it, as G. Place the card in a corresponding position on the drawing and mark the point G. Join FG, and produce the line to meet the edge of the sketching block in J.

Proceed, in the same manner, to find the receding line FK, and produce it to meet the edge of the sketching block in N.

If we were to produce the lines FG and FK to meet the line HL (the horizon), also produced, we should obtain the vanishing points for lines parallel to FG and FK respectively; but as it is advisable, when sketching from nature, to restrict all our construction to the limits of our sketching block, we will deter-

mine the perspective representation of the other receding lines in the following manner :—

Divide the line EF into any number of equal parts (4); also the line HN and the line LJ into the same number of equal parts, and join the corresponding points. These divisions may be repeated on each of the lines, as well as below HL, and the corresponding points joined, as shown.

These lines converge towards their respective vanishing points on the horizon line, and form guiding lines for drawing the receding lines in the sketch. The divisions on the vertical lines could be subdivided if necessary, and the corresponding points joined by lines, so as to form guiding lines nearer the lines we wish to draw; they could also be determined by the construction shown in Prob. 44.

To determine the various widths of the buildings, place the sketching block in a vertical position at the end of the string, as before, with the point A covering the nearest angle; and mark off upon its upper edge the several widths, as R and S, and draw vertical lines from them, as shown (Fig. 165).

To determine the inclination of the lines receding in an opposite direction, *e.g.* the ridge of the nearest roof on the right, select one of the guiding lines receding to the left, at a convenient height, and produce it, as PQ.

If we wish to take less of the subject in our sketch, we should, of course, take up our position nearer the portion we wish to draw.

We will now show how these lines could be determined without using the piece of string or the card.

In any convenient position on our sketching block, let us draw a vertical line *af* (Fig. 166), to represent as much as we can see of the nearest angle of the nearest building. The length of this line may be selected at pleasure, and will depend upon the size we wish to represent the building.

Hold a pencil horizontally, at arm's length, and as near the level of the eye as we can judge, and notice where it appears to

cut the line *af*, and mark the point in a corresponding position on our drawing, as *e*. Draw a horizontal line HL through *e* for the horizon line. Hold the pencil horizontally at arm's length again, and measure the apparent distance between the vertical lines on the right, as *bg*. Turn the pencil vertically and compare this measurement with the line *af*. Let us assume it to measure one half of *af*. Set off this proportion horizontally, to the right of *af* on the drawing, as *bg*, and draw a vertical line through *g*. Hold the pencil horizontally, as before, on the point *g*, and notice where it appears to cut the line *af*, as *b*. Mark a corresponding point *b* on the drawing, and draw a horizontal

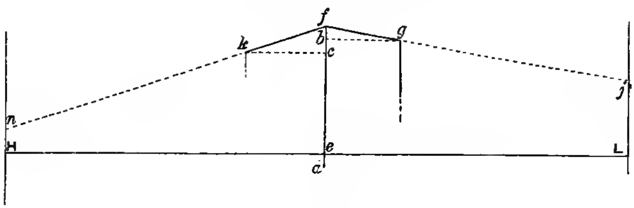


Fig. 166

line from it to cut the vertical line on the right in *g*. Join *fg*, and produce the line to meet the edge of the sketching block in *j*. Determine the line *fn* on the other side of the building in a similar manner. Divide the lines *ef*, H*n*, and L*j* into a corresponding number of equal parts, and join the respective divisions by lines, as shown in Fig. 165. The other parts of the building can be determined by holding a pencil at arm's length, taking the required measurements, and comparing them with the line *af* in the building, and then setting them off to a corresponding proportion of the line *af* on the drawing.

This method of finding the main lines of an object in perspective is somewhat more difficult for the beginner than the former method, because it is necessary for him to be able to judge, with some degree of accuracy, proportions by the eye, *e.g.* the proportion of *fb* to *fa* (Fig. 166), which is not necessary in the former method; so it is advisable for the

student first to practise the former method until the eye has acquired some experience in estimating these proportionate measurements.

The second method has many advantages; for instance, the object can be drawn to any scale from one position, *i.e.* without changing our distance from it; the work can also be accomplished in less time.

CHAPTER XIII

AËRIAL PERSPECTIVE

Aërial Perspective (Lat. *aër*, air) is the art of representing in a drawing the comparative distances of objects by their relative depth of tone or distinctness.

The atmosphere contains, in a state of suspension, small particles of matter and globules of water, which interfere with its transparency; consequently, the more distant an object is from the eye—providing the atmosphere extends from the eye to the object—the less distinctly is it seen.

We are enabled, by linear perspective, to judge the comparative distances of objects, of known dimensions, by their proportion to other objects shown in the drawing, or by the arrangement of their principal lines; but there are certain objects in nature, such as mountains, hills, etc., the relative proportion of which to each other, or to the other objects shown, may not be known; the relative distances of these objects can be indicated by aërial perspective.

Let us imagine we are looking out at the sea, on which three vessels, *a*, *b*, and *c*, similar in colour, are sailing, and let the nearest vessel (*a*) be half a mile distant, the next vessel (*b*), one mile, and the farthest vessel (*c*), one and a half miles, all measured in the direction of sight. Now if we represent the depth of tone of *a* by 6, *b* should be represented by 4, and *c* by 2. In other words, there should be as much difference of tone between *b* and *c* as between *a* and *b*. Of course we assume

the atmosphere to be homogeneous, *i.e.* containing the same amount of suspended matter throughout the distances specified, which is not always the case.

So far as relative tone only is concerned, there is very little theory to be learned with reference to aërial perspective; it is essentially a practical subject, and its effects can only be correctly reproduced from accurate observation of nature, but the *effect of aërial perspective as regards colour* admits of a more theoretical explanation, and there are certain laws in reference to it, resulting from the discoveries of Professor Tyndall and Lord Rayleigh, the knowledge of which will greatly aid the student in reproducing these effects.

Aërial Perspective in Relation to Colour.—We have already mentioned (page 12) "that our knowledge is apt to interfere with our judgment in the appearances of objects." This also applies specially to aërial perspective; there is a constant tendency for the uninitiated to reproduce the appearances of distant objects in the colour they are known to possess when placed in the foreground, whereas, in several instances, their colour is entirely different, especially if the distant object is of a dark colour.

We are conscious of the existence of objects by means of the senses of sight, hearing, and feeling, and these senses are excited by means of vibrations that are communicated to the optic, auditory, or sensory nerves. When a wire in a state of tension is struck it vibrates, and if there is sufficient tension the vibrations will produce a musical note. If the tension on the wire is increased the vibrations become quicker, and the musical note higher in tone. These vibrations impart a mechanical state of vibration to the air, which is communicated to the tympanum of the ear, and through the mechanism of the ear to the auditory nerves. If the vibrations are less than 16 to the second no sound is heard, but if between 16 and 38,000 per second they are audible; again, the ear is incapable of receiving any impression from vibrations of greater frequency.

With reference to sight and the sensation of heat, impressions are communicated by means of undulations in the ether that pervades space. Waves are generated in this ether by means of a source of heat or light. The heat impressions are transmitted through the sensory nerves, and those of light through the optic nerves.

The vibrations of sonorous waves move in the direction of their paths, while the heat and light rays vibrate transversely to their paths.

If we admit a ray of sunlight into a darkened room through a small hole in a shutter, as shown in Fig. 167, S being the shutter and A the ray of light, and if we intercept the ray by a

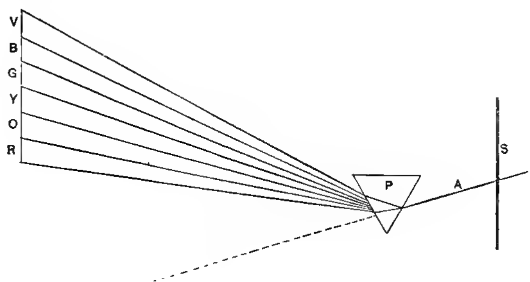


Fig. 167

triangular prism of glass (P), placed so that the ray of light shall strike one of its faces obliquely, as shown, the ray of light on passing through the prism will be refracted and dispersed. If the light, after passing through the prism, fall on a white screen placed vertically at about 5 yards from the prism, it will form a band of coloured light with colours arranged in the following order: viz. red, orange, yellow, green, blue, and violet, as shown by corresponding letters in Fig. 167.

This band of colours, which is seen, is known as the solar spectrum, and the colours are similar to those seen in the rainbow; they are also called the prismatic colours from the fact that they can be produced by a prism of glass.

These colours are produced by ether waves of different

lengths moving with marvellous rapidity, *e.g.* red, which is the least refrangible colour in the spectrum, has a wave length of about $\frac{1}{33000}$ of an inch, and 390 billions of waves succeed each other in a second of time; while violet, which is at the other end of the spectrum, has a wave length of $\frac{1}{65000}$ of an inch, and is produced by undulations at the rate of 770 billions in one second; the intermediate colours are produced by waves varying in length between these extremes.

If the ether waves are more than the $\frac{1}{33000}$ of an inch apart, or take longer than the $\frac{1}{390}$ billionth of a second to follow each other, heat rays only are produced; while, on the other hand, if they are less than the $\frac{1}{65000}$ of an inch apart, and take less time than the $\frac{1}{770}$ billionth of a second to pass a certain point, chemical or actinic rays only are produced.

We will now consider how the lengths of these different waves affect the appearance of colours in aërial perspective.

If we add a few drops of milk to a tumblerful of pure water, it will appear bluish or cold in colour if seen by reflected light, because the particles of milk are large enough to intercept the shorter waves, *i.e.* the blue and violet, and disperse them, reflecting some back towards the eye, while they have less power over the longer waves, the red and orange, several of which pass on through the liquid; but if we hold the tumbler between the eye and the light, so as to allow the light to pass through it to the eye, it will appear warm in colour, because, as before, most of the shorter waves are dispersed, while a great number of the longer waves succeeds in passing through the liquid to the eye.

If we gradually add more milk to the solution, while still holding it between the light and the eye, we shall, of course, obscure more of the light that passes through it, and we can, by this means, extinguish the waves of each colour in succession. Any blue and violet rays that may have succeeded in passing through the weaker solution will be first of all cut off, then the green, yellow, orange, and, lastly, the red, and if we still increase the opacity of the liquid these waves also will be extinguished.

Now a similar effect to this occurs when sunlight passes through the atmosphere. The glass of water may be taken to represent the transparent air and the milk the particles held in suspension. Let us imagine we are looking at a morning or evening sky with the sun near the horizon. If the sky is clear and the weather somewhat hazy, the effect we wish to study will be more decided. On looking directly above our heads, towards the zenith, we see a blue sky, because the blue and violet waves are reflected towards us, while the longer heat waves pass on. If we now pass our eyes gradually across the sky towards the horizon, we shall find the colour of the sky appears gradually to get warmer, because, as the light passes through more of the denser portion of the atmosphere, more of the longer heat waves in proportion penetrate to us, and when we look towards the horizon, all the waves are dispersed with the exception of the red.

This is called the effect of *turbid media*, and may be summed up as follows :—

Exceedingly small particles of matter are incapable of transmitting white light in its pure state, but disperse the shorter waves, blue and violet, more than the longer waves, red and orange. Consequently, the residue of those waves that have escaped dispersion alone is transmitted, consisting principally of the red and orange, and the reflected colour is bluish or cold in appearance.

Let us try another experiment. If we look at a piece of black material, such as ebonite or glass, at a certain angle, so as to avoid reflected light, it will appear black or nearly so, because most of the light waves are absorbed. A drop of milk placed on its surface will appear blue, because most of the shorter waves are arrested and reflected, while the longer waves pass through the fluid and are absorbed by the black material. As the drop of milk is translucent, we are enabled, to a certain extent, to see the black material through it, which intensifies the bluish appearance. As we have already noticed, when looking towards

the zenith, space which would otherwise appear dark, appears blue when seen through the lighted atmosphere.

Distant mountains in shadow, as well as the shaded side of distant trees, also appear bluish-violet in colour for the same reason.

If sunlight is reflected from distant objects, most of the shorter waves, green, blue, and violet, will be intercepted and dispersed, while most of the longer waves, yellow, orange, and red, will be transmitted to the eye. This effect is more marked when the sun is near the horizon, as the light waves then pass through more of the denser layers of the atmosphere. Distance has very little effect upon light reflected from a white object. We may sum up these results as follows :—

All dark objects seen through the atmosphere at a distance appear lighter and bluer in colour, while all light objects appear warmer, and, if very light, or white, they are very slightly affected by distance.

When the weather is misty, the particles of water held in suspension are too large to make any appreciable difference in the dispersion of the various wave lengths, consequently both transmitted and reflected light reach the eye unchanged in colour, although very much weakened.

The principle of turbid media also affects pigments. If we take a little yellow ochre and brush it thinly over a white ground, it appears a rich orange colour, because the light reflected from the white surface, on passing through the finely divided particles of the pigment, becomes sifted; most of the blue and violet waves are intercepted and dispersed. Consequently, the orange colour is the result of a mixture of the longer waves that have succeeded in passing through with the colour of the pigment.

Again, if we try a similar experiment with yellow ochre on a black ground, the result will be a greenish colour, because the finely divided particles of the pigment are too small to intercept many of the longer warm waves which succeed in

passing through to the black surface underneath, where they are absorbed, while the shorter waves are intercepted and dispersed; many of them on being reflected to the eye mix with those forming the tint of the pigment, thus forming the greenish colour.

The same principle applies to all pigments, from which the following law may be deduced:—

Any pigment when laid thinly upon a light ground appears warmer in colour, and cooler when laid upon a dark ground.

It is not within the province of this work to go into the harmony of contrast in colours, but we may here state that those beautiful harmonies of colour that exist in nature, and which are seen to the greatest perfection in mountain scenery when the sun is near the horizon, can be explained by the laws of aërial perspective.

The greatest contrast between two colours exists when they are *complementary* to each other.

COMPLEMENTARY COLOURS.—We have shown that white light can be separated into several distinct colours, and if these colours were combined, they would produce white light. It is not necessary to unite the whole of them to produce this result; we may select colours from different parts of the spectrum, and by combining them produce white light, but it would not be so bright as if the whole number in the spectrum were employed for this purpose. If we select three such colours, by combining any two we obtain a tint of which the third is said to be complementary.

There are an immense number of complementary colours, from which the following pairs may be selected: red and blue-green, orange and greenish-blue, greenish-yellow and violet-blue.

Now it will be noticed that one of the colours in each pair is formed by long waves, while the other is produced by short waves; and this explains why it is that complementary colours are seen when sunlight and shade are seen upon objects in the distance.

It is generally acknowledged by artists, that the harmony caused by the contrast of complementary colours is a little too severe to be pleasing, and that the harmony is more satisfactory if we blend one or both of the colours with a little gray. Now this is precisely what happens in nature, as already shown by the laws of aërial perspective.

CHAPTER XIV

PERSPECTIVE HINTS FOR ARTISTS

Note.—Numerals in brackets [ ] refer to the numbered articles in Chapter II.

In portrait painting, the position of the *horizon line* [18] is one of the most important points for an artist to consider before commencing a picture.

A figure drawn with a low horizon line has a more important appearance than one drawn with a high horizon. We intuitively associate the position of the horizon line with the level of our eyes. Consequently, if the head of a standing figure represented in a picture is above this line, it suggests to us, when we look at the picture, a figure taller than ourselves; while a similar figure shown with a high horizon line has the contrary effect, of one on which we are looking down.

The old portrait painters took great liberties with the position of this line; Gainsborough is a striking example; his backgrounds are generally painted with the horizon line in a much lower position than the level of the eye from which the figure was drawn, and as the horizon line of the background is the one the eye naturally follows, when looking at the picture —being, in fact, the only one shown—his figures appear to have colossal proportions.

When a figure is drawn "life size," it has a more natural appearance if the position selected for it is not too far from the eye, as the perspective of the curves of the features is more

apparent when the subject is near. Now the artist is confronted with the difficulty, that the nearer the figure is placed to the eye, the less of it will be comprised in the *field of vision* [44]. To overcome this difficulty it is usual, when working in a studio, to place the model on a raised platform, about 12 inches high, called "the throne."

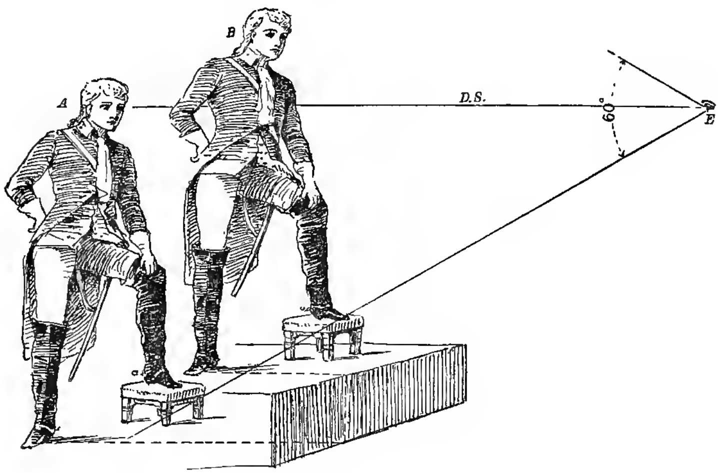


Fig. 168

In Fig. 168, A is a figure standing on the *ground plane* [2], and E is the position of the spectator's eye.

To insure the figure (A) being within the *angle of distinct vision* (60°), it is necessary to place it as far as the distance EA from the eye. Now this distance would be too far to give the proper relief and natural appearance to the perspective of the curves of the features; so to bring the figure closer to the eye, and at the same time to include the whole of it within the angle of distinct vision, it is necessary to raise it above the level of the ground plane, as shown by the figure B (Fig. 168).

This not only gives the figure a more important appearance, as already described, but has the additional advantage of subordinating the horizontal plane, upon which the figure is stand-

ing, as it recedes more rapidly, and consequently occupies less space in the picture than it would if shown in connection with the figure placed at A.

It will be seen from this illustration (Fig. 168) that in drawing a full-length figure of an adult "life size," the artist, to overcome a perspective difficulty, has to resort to a lower horizon line than that which generally accompanies the figure, and, although it has the advantage of giving the figure a more important appearance, as well as enabling the artist to show more clearly those curves of the features that are nearly horizontal, still it somewhat detracts from the natural appearance of the person in the portrait. So it is advisable not to place the horizon line lower than is absolutely necessary to meet the requirements of the angle of distinct vision [44].

When the portrait is only "half length" the same difficulty has not to be met.

In painting the portraits of children, they should be represented with a high horizon line to appear natural, as the eye of the spectator is generally above their heads; and as they are much smaller than adults, it is possible to include them in the angle of distinct vision even though they be placed comparatively near the spectator. The horizontal curves of the features being below the level of the eye, it will be possible to show them in perspective with the same facility as if they were placed above it.

Fig. 169 shows how the relative size of a figure in a picture can be determined in proportion to a given figure, and its distance from the picture.

Let A be the given figure, and H.L. the horizon line. Assume any point V on the H.L. for a *vanishing point*, and draw lines from the extremities (B and C) of the figure A to it. Now these lines represent, in perspective, receding parallel lines [13], and form a *perspective scale* (p. 50) for objects of a given height, *i.e.* that of the figure A.

Suppose we wish to draw, standing at D, a figure of the

same height as A. Draw a horizontal line from D to meet the line CV in E. Draw a vertical line at E to meet the line BV in F. Then EF is the height of the figure required. In the same way HI is the height of a similar figure standing at G.

By producing the lines forming the perspective scale towards

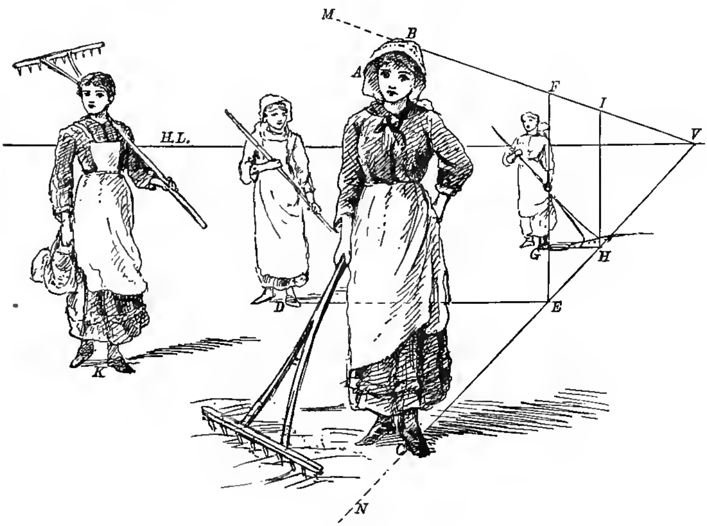


Fig. 169

the spectator, as shown at N and M, we could determine, in the same manner, the height of a similar figure placed nearer the spectator than the figure A.

It should be noticed that the horizon line intersects each figure in a similar position, *i.e.* about three-quarters of its height, from which we may infer that the eye of the spectator is at a lower level than the eyes of the figures represented, and as the level of the ground plane below the height of the spectator's eye is constant for any distance from the picture [19], it naturally follows that the H.L. must intersect each figure at a similar height. By bearing this in mind it will be possible to draw figures at any distance from the picture, and determine their relative heights, without using the perspective scale ; *e.g.* let us

draw a figure standing at K, similar in height to the given figure A. We know that the distance of K below the H.L. is equal to three-quarters of the height of the figure we wish to draw. Consequently, if we take one-third of this distance and set it up above H.L., it will give us the top of the figure, as shown.

We will carry this method a step farther in the following illustration.

Note.—If we wish to draw a figure taller or shorter than the given figure at a selected distance from the picture, we could easily determine the necessary proportion if we first determine the perspective height of a figure similar to the given figure at the distance required.

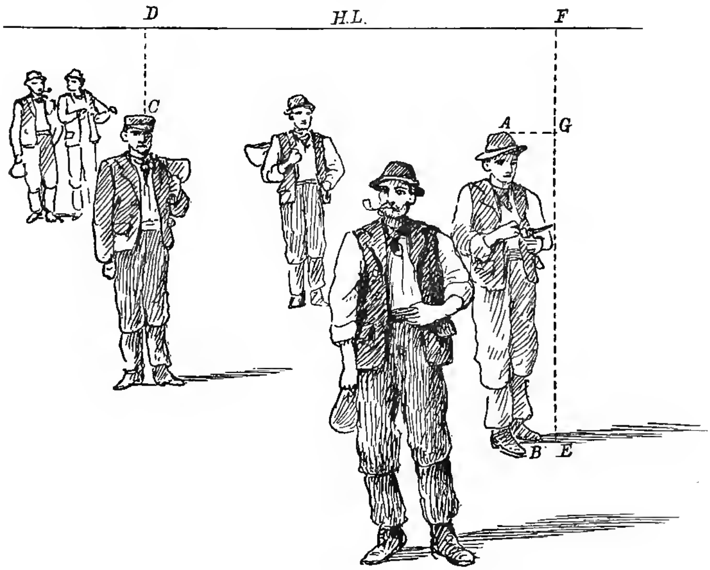


Fig. 170

Let C (Fig. 170) be a figure, given in position in the picture, and H.L. the horizon line. We wish to draw other figures of a similar height at different distances from the picture. On measuring the distance of the top of the head of the figure from the H.L., we find it is equal to one-third the

height of the figure; this proportion is constant for every figure of a similar height in the picture, *e.g.* let us draw a figure of a similar height standing at B. Divide the height EF into four equal parts. Then the height EG of the lower three divisions represents the perspective height of the figure. The heights of all the other figures shown could be determined in a similar manner.

The perspective heights of figures taller or shorter than the given figure could be determined in a similar manner to that described in the previous illustration.

We will now proceed to show how to draw in perspective

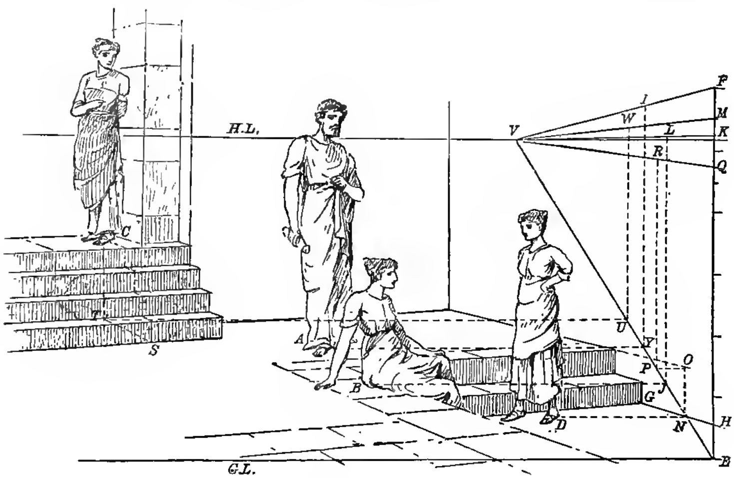


Fig. 171

figures of given heights on horizontal planes at different levels.

Let G.L. (Fig. 171) represent the *ground line* [4], and H.L. the horizon line.

At any point E on the G.L. draw a vertical line EF, divided into six equal parts, and let this represent a scale of six feet.

We wish to draw a figure standing at A, six feet in height.

Assume any point V on the H.L. for a vanishing point, and draw lines from E and F to it, to form a perspective scale for six feet.

Draw a horizontal line from A to meet the line EV in Y. Draw a vertical line from Y to meet the line FV in I. Then YI is the height of the figure required.

We will now draw a figure whose height would be five feet three inches if standing on the ground plane at B. Draw a line from K, five feet three inches above E, to V. Draw a horizontal line from B to meet the line EV in J. Draw a vertical line from J to meet the line KV in L. Then JL would be the height of the figure if standing at B, which will give us the proportions for drawing the sitting figure.

Let us now draw a figure five feet six inches high, standing at C. First determine the position of a point on the ground plane immediately below C; this is found by drawing a line from any vanishing point on the H.L., through C, to meet the edge of the step, and then drawing a vertical line from the point thus found to the level of the next step, and proceeding in the same manner with each step till the level of the ground plane is reached in S. Draw a line from S towards the same vanishing point, to meet a vertical line from C in T.

Draw a line from M, five feet six inches above E, to V. Draw a horizontal line from T to meet EV in U. Draw a vertical line at U to meet the line MV in W. Then UW is the height of the figure standing at C.

Let the next figure be four feet nine inches high, and standing at D. Draw a line from Q, four feet nine inches above E, to V. Draw a line from D to meet the line GH in N. Draw a vertical line at N to meet the level of the ground plane produced in O. Draw a horizontal line from O to meet the line EV in P. Draw a vertical line at P to meet the line QV in R. Then PR is the height of the figure standing at D.

To draw Boats in Perspective.—A boat, foreshortened, is one of the most difficult objects to draw correctly, as it is composed of so many *lines of double curvature* (p. 193). A knowledge of how the laws of perspective may be applied to this purpose will

greatly assist the artist in drawing a boat of any given dimensions, and in any desired position. This knowledge will also enable him to correct a sketch made from an existing boat.

Let A and A′ (Fig, 172) represent the plan and elevation of

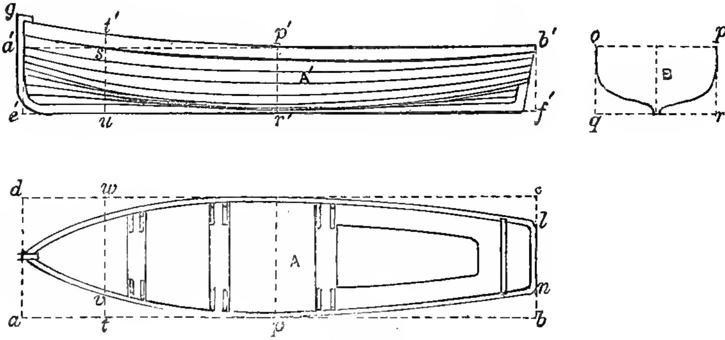


Fig. 172

a boat; and let *abcd* be the plan, and *a′e′f′b′* the elevation of a rectangular solid. Now we can draw a rectangular solid, in perspective, to any given dimensions, and in any given position, as already shown in the preceding portion of this work. Let the

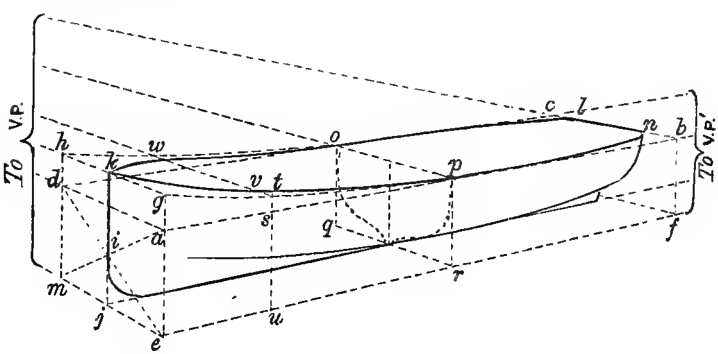


Fig. 173

corresponding letters in Fig. 173 represent such a solid drawn in perspective. The left-hand end of the boat—the bow—(Fig. 172), is much higher than the opposite end—the stern—*i.e. a′g*

is about one-third of *a′e′*. Make *ag* (Fig. 173) one-third of *ae*, and draw a line from *g* to V.P. to meet the vertical line at *m* in *h*. Draw the diagonals *am* and *de*, and draw a vertical line *kj* through their intersection. Draw the curves *gp* and *ho* to correspond with the curve *gp′* (Fig. 172). The width of the stern *ln* (Fig. 172) is about three-fifths of the line *cb*. Mark off the same proportion on the line *cb* (Fig. 173), as *ln*. Any number of points in the curve forming the upper part of the boat—the gunwale—may be determined as follows: The distance *a′s* (Fig. 172), is one-third of *a′p′*. Draw a vertical line in a corresponding position in Fig. 173, as *tu*, and draw a line from *t* to V.P. Set off *tv* (Fig. 173), bearing the same proportion to *tw* as in Fig. 172. Of course all the distances in Fig. 173 must be set off perspectively. Draw a fair curve through the points *n*, *p*, *t*, *k*. Find corresponding points on the opposite side of the boat, and proceed in a similar manner. B (Fig. 172) is a cross section through the middle of the boat; this, as well as any additional cross section, can be drawn in perspective, as shown in Fig. 173, by Prob. 34. These will enable the artist to determine the other curves of the boat.

Fig. 174 shows the boat completed. All the straight lines

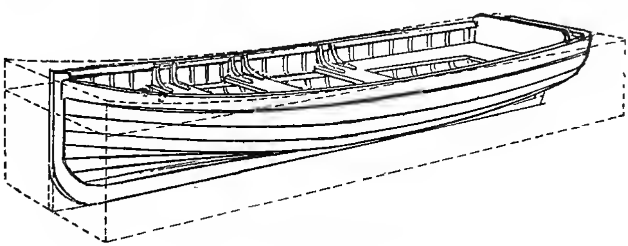


Fig. 174

receding to the left converge towards V.P., while all those receding to the right converge towards V.P.′, and if the boat were floating in calm water, these vanishing points would be on the horizon line [20].

Fig. 175 shows the same principle applied to a boat in

oblique perspective. If it were resting on level ground, V.P.′ would be on the horizon line [20]; and if the stern of the boat were in a vertical plane, A.V.P. and A.V.P.′ would be on the same vertical line [36].

This method of drawing boats will be found very useful in practice, but the artist should first endeavour to acquire facility in sketching rectangular solids in any position in perspective.

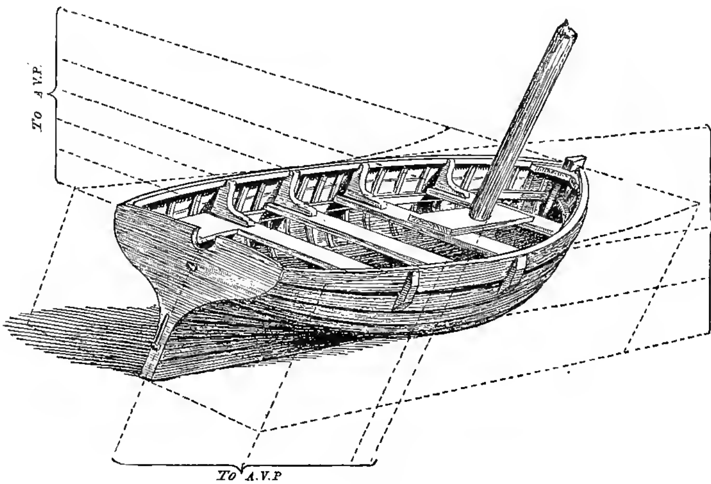


Fig. 175

He will then find, after a little practice, that the delineation of the difficult curves of a boat will be greatly simplified. The lines of the solid will form a guide, by means of which the proper direction of the curves will be apparent. It will also enable him to correct, by the laws of perspective, sketches of boats he may have made.

To determine the relative sizes of objects according to their distances.

Let A (Fig. 176) be a boat drawn in the picture; we wish to draw similar boats, in the same picture, at different distances. Draw a horizontal line at the water line of the boat,

and in any convenient position on this line, d for instance, draw a vertical line equal in length to the total height of the boat, as de. Assume any point v on the H.L. for a vanishing point, and draw lines from d and e to it. These represent, in perspective, parallel lines [13].

Note.—H.L. is the perspective horizon [18].

We now wish to ascertain the relative size of a similar boat

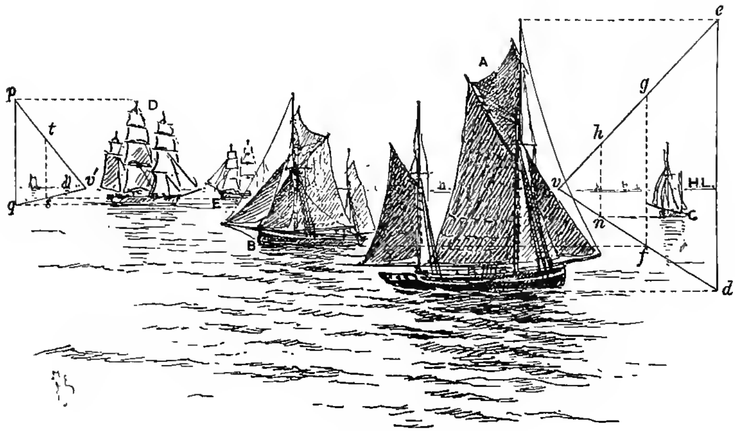


Fig. 176

placed at B. Draw a horizontal line from B, to meet the line dv in f. Draw a vertical line at f to meet the line ev in g. Then fg is the height of the boat required.

The line dv is at the sea level, and the lines dv and ev form a perspective scale (p. 50), for objects of the same height as the boat A, and if we ascertain the height of a similar boat corresponding to its distance, we can draw the rest of the boat in the same proportion; *e.g.* C is a similar boat, foreshortened; its height, hn, is determined as already described.

D is a much larger class of vessel—a barque—and if we wish to repeat this in the picture, we must construct, in any

convenient position, a perspective scale for its height, as pqv', as already described for the boat A. And if we wish to draw a similar vessel at E, st is its height.

If the lines forming the perspective scales were produced towards the spectator, we could determine, if necessary, the relative heights of objects nearer than the given objects.

CHAPTER XV

PERSPECTIVE FOR ARCHITECTS AND ENGINEERS

Note.—Numerals in brackets [ ] refer to the numbered articles in Chapter II.

The method of perspective used by architects and engineers differs somewhat from that illustrated in the previous portion of this work. Plans and elevations of the object are generally made first, and from these the perspective drawing is constructed by finding where visual rays [9], from various parts of the object, will intersect the picture plane [3].

On the Position of the Picture Plane.—Let A (Fig. 177) be the plan of a building, and S the station point [11].

It has been shown [16] that the position of the picture plane depends entirely upon the direction of sight; so we must first determine the direction in which we wish the spectator to be looking, before we can find the position of the picture plane. One method adopted by architects is as follows :—

Draw lines from the corners *a* and *b* to S. Bisect the angle *a*S*b* by the line S*c*. Draw a line *pp* through the corner C at right angles to the line S*c*. Then *pp* is the position of the picture plane.

Although by this method it is possible, especially when the building is much longer on one side than the other, to place the spectator nearer the building, if desirable, than by any other method, and at the same time include the whole of it in the field of vision [45], still, the appearance of the perspective

is not so pleasing as is the case when the following method of placing the picture plane is used.

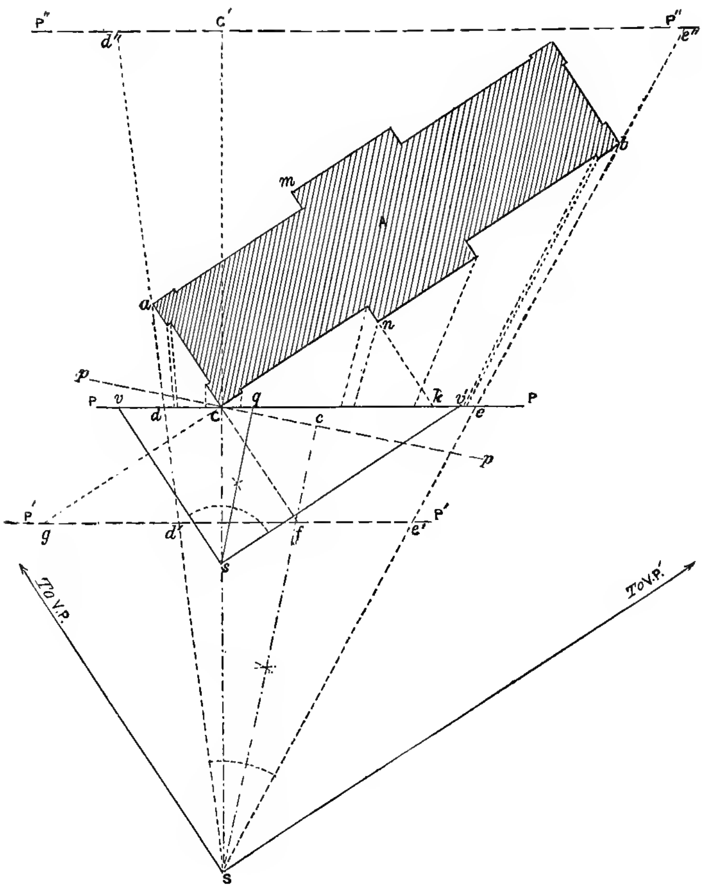


Fig. 177

Join CS. Draw a line PP through C at right angles to SC. Then PP is the position of the picture plane.

By placing the picture plane in contact with the corner C of the building, we simplify the work, because a vertical line on the picture plane at C serves for a line of heights [46],

but by so doing we restrict ourselves to one scale for the perspective drawing.

If we wish to make the drawing smaller, we must place the picture plane nearer the position of the spectator (S); *e.g.* if the picture plane is placed in the position PP, *de* will be the perspective width of the building; but if we place it at P′P′, *d′e′* will be its perspective width. We should then have to produce the line *a*C to meet P′P′ in *f* to determine the position of the line of heights. Or the line *b*C produced to meet P′P′ in *g* would answer the same purpose.

It is possible to make a drawing larger in scale than the plan, *e.g.* draw a line P″P″ at right angles to SC produced to C′. Then *d″e″* would be the perspective width of the drawing. This method is called *stereographic projection,* because the object, instead of being seen through a plane by perspective (*perspicio*), is projected as a solid (*stereos*) on to a plane behind it. As this is done by *radial projection,* the appearance of the drawing is precisely similar to its perspective appearance.

Supposing now that it is decided to have the picture plane in the position of the line PP (Fig. 177). Lines drawn from the various angles of the building to S will determine the perspective widths of the several parts on the line PP.

We will now determine the vanishing points.

A line drawn from S, parallel to the side of the building C*b*, to meet the line PP produced, will determine the vanishing point (V.P.′), for all lines parallel to C*b*; and a line drawn from S, parallel to C*a*, will in like manner determine the vanishing point (V.P.) for all lines parallel to C*a*. It is more convenient, as a rule, to find these vanishing points to a smaller scale, as already described (p. 65), *e.g.* take any aliquot part of SC, say one-third, as *s*C, and draw lines from *s*, parallel to each side of the building, to find *v* and *v′*. We can then repeat the distances C*v* and C*v′* three times on each side of C, in the perspective drawing, to find V.P. and V.P.′

In addition to these points it is a great advantage to have a

diagonal vanishing point [43]. This is found by bisecting the angle *vsv′* by the line *sq*. Then *q* is the point required. In the perspective drawing we, of course, repeat the distance C*q* three times.

It is also advisable to have a line of heights for the central portion of the building. Its position can be determined by producing a line through the points *mn* to meet the line PP in *k*. We could also, in the same manner, determine the position of any other line of heights we may require—the ends of the roofs, for instance.

Fig. 178 represents a plan and elevation of the same

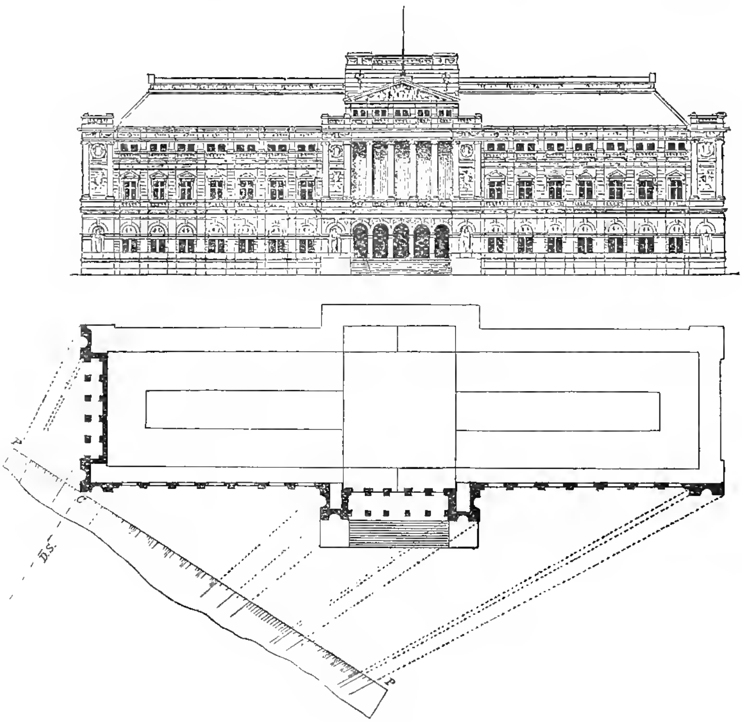


Fig. 178

building shown in Fig. 177, but drawn to a larger scale. PP is a strip of paper, the straight edge of which is placed in a

corresponding position to the line PP (Fig. 177), and lines are drawn on it from the several angles of the building to a corresponding station point. The longest lines on the strip of paper mark where the visual rays from the principal angles of the building intersect the picture plane, the shorter lines the angles of the piers, and the shortest lines the window openings, etc. C is the centre of vision [14], and D.S. the direction of sight [15].

Fig. 179 is the perspective drawing, which is constructed as follows :—

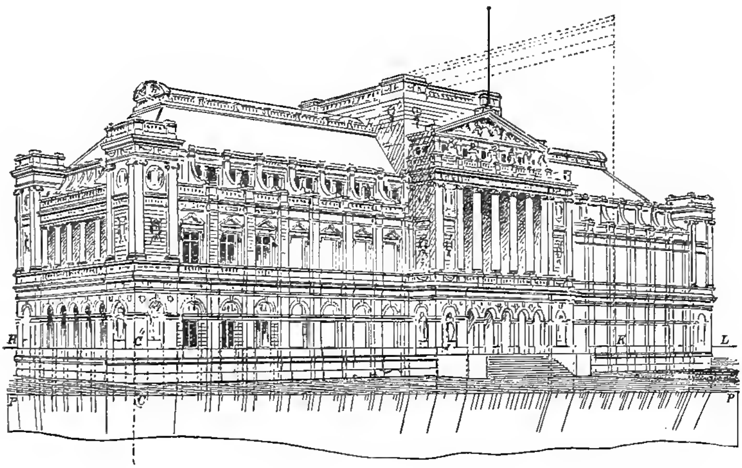


Fig. 179

Having selected the position of the line HL (horizon line), to represent the level of the eye, place the strip of paper in any convenient position, with the edge PP below HL and parallel to it.

Note.—To show the detail more clearly this is drawn to a larger scale than it is shown in Fig. 178.

Draw a vertical line from C to meet HL in C′ (the centre of vision). Set off on each side of C′, along the horizon line, the necessary distances for finding the vanishing points, etc., as already described (p. 263).

Draw a vertical line through C′ for a line of heights, and mark off upon it the necessary heights from the *elevation*, and draw lines from them to each vanishing point.

Note.—If this drawing were made to the same scale as Fig. 178, the heights could be set off at once from that drawing.

Draw vertical lines from the lateral divisions marked on the line PP, to determine the corresponding widths in the perspective drawing.

All horizontal lines receding to the left are drawn towards V.P. (the left-hand vanishing point), while all those receding to the right are drawn towards V.P.′ (the right-hand vanishing point).

The vertical line at K is the line of height for the central portion of building, the position of which is fixed by *k*, as shown in Fig. 177. The necessary heights are set up on this line and then drawn towards the vanishing point on the left to meet the angle of the building, as shown.

Large buildings look more stately, and have more repose, when the station point is taken at a great distance, so that the whole building can be included in an angle of from 30° to 40°. The angle selected for Fig. 179 is 35°.

Small buildings, or those of a picturesque character, look better when a larger angle is taken, *i.e.* from 50° to 60°.

Buildings always looks more imposing when they are placed on an eminence, that is when seen from a low horizon, for the same reason as already explained at the commencement of Chapter XIV.; and many architects, to give their designs a better appearance, make the foregrounds of their perspective drawings ascending planes [38]. But this device should never be resorted to when the building represented is to be erected in a locality that is really horizontal, as it is very misleading to those unacquainted with the artifice, and is likely to cause disappointment after the building is erected.

To draw Mouldings in Perspective.—Let B (Fig. 180)

be the section of a moulding we wish to draw in perspective at the top of the pedestal A.

Enclose the section of the moulding within a rectangle and divide it into any number of similar rectangles. Draw these rectangles, with their divisions in perspective, in any convenient positions, as shown at D and E, and draw curves through the corresponding divisions of each rectangle, similar to those

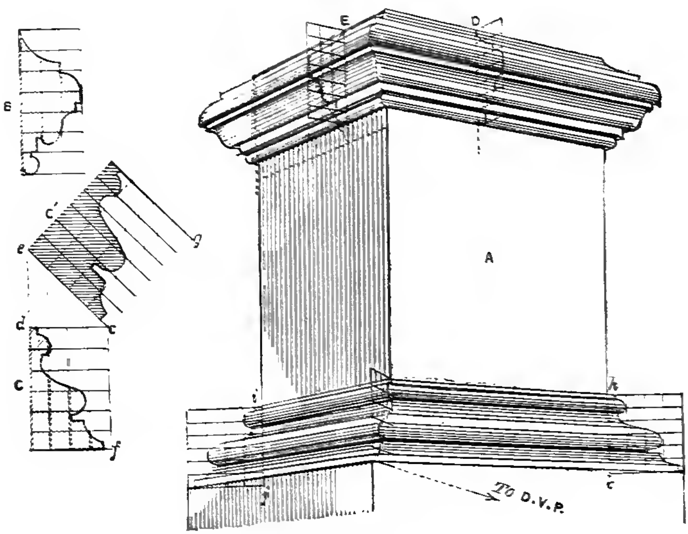


Fig. 180

shown in B. These curves form the profiles of the mouldings in perspective. Lines drawn to each vanishing point, through the various points of these profiles, give us the mouldings in perspective, as shown.

When the same moulding is carried round many angles, as shown in Fig. 180, it is better first to draw the moulding in perspective at the angles, in the following manner.

Let C be the section of a moulding we wish to draw in perspective round the base of the pedestal A. First enclose the section in a rectangle, and divide it into any number of equal rectangles, as before. Draw a line from *c*, at an angle of

45°, to meet a vertical line at *d* in *e*. Draw *cg* at right angles to *ce*, and equal in length to *cf*. Divide *cg* into the same number of equal divisions as *cf*, and draw lines from each division, parallel to *ce*. Draw vertical lines from each division on *cd* to meet *ce*, and from these points draw lines parallel to *cg*. Draw curves through the corresponding divisions of each rectangle, similar to those shown at C. Then C′ represents the section of the moulding at the angles of the pedestal A.

Draw the rectangle C′, with its divisions in perspective, at each corner of the base of A. The nearest one would be more easily drawn by means of a diagonal vanishing point [43]. The horizontal lines forming the divisions of the other two corners could be determined as follows. Having determined the heights *hk* and *ij*, divide each into the same number of equal divisions as *cg*. Lines produced through the corresponding points at each corner will determine the horizontal lines required. Draw curves through the corresponding divisions of each rectangle in accordance with C′, and complete the drawing, as shown.

When it is required to draw mouldings round cylindrical surfaces, or circular openings, the profile should be first drawn, in each instance, by perspective, on planes passing through a common axis, as illustrated in Prob. 35.

TO DRAW GROINED VAULTING IN PERSPECTIVE.—Let A, B, C, D, (Fig. 181) be the angles supporting the groins, and EHJ the form of the arch of the intersecting vaults.

Draw a horizontal line EJ, and a vertical line from H to meet it in *c*. Divide E*c* into any number of equal parts (3), and draw vertical lines from each division to meet the arch in *d* and *e*.

Join AC and BD, intersecting in F. Draw a vertical line at F, to meet a line drawn from H to the centre of vision in G. Produce CA to meet JE produced in N. Draw a vertical line at N, to meet horizontal lines drawn from *d* and *e* in K and L. Draw lines from *a* and *b* to the centre of vision, to intersect the lines AC and BD in the points *fg* and *jh*. Draw lines from L

and K to P.D. to meet vertical lines on *g* and *f* in the points *l* and *k*. Draw a fair curve through the points A, *k*, *l*, G.

Draw horizontal lines from *e* and *d* to meet the opposite side

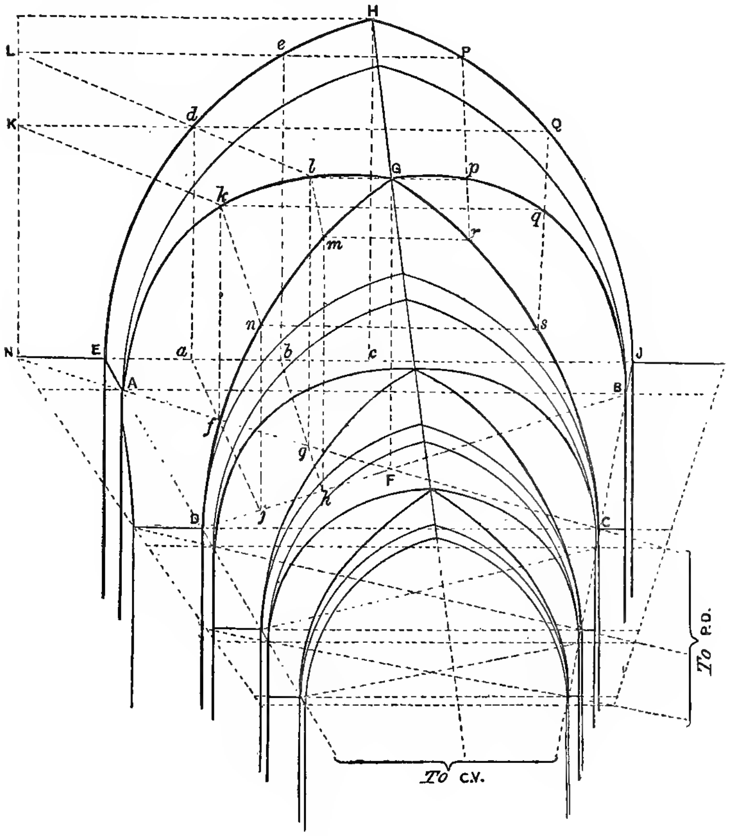


Fig. 181

of the arch in the points P and Q. Draw lines from P and Q to C.V. to meet horizontal lines drawn from *l* and *k* in the points *p* and *q*. Draw a fair curve through the points B, *q*, *p*, G. Draw vertical lines at *j* and *h* to meet lines drawn from *k* and *l* to C V. in the points *n* and *m*. Draw a fair curve through the points D, *n*, *m*, G. Draw horizontal lines from the points *m* and *n*

to meet lines drawn from p and q to C.V. in the points r and s. Draw a fair curve through the points C, s, r, G. Proceed in a similar manner with the remaining groins and complete the drawing, as shown.

To draw Gothic Tracery in Perspective.—The geometrical drawing of the tracery should first be squared, as shown in the geometrical drawing in Fig. 78. The squares should then be drawn by perspective in the position required, and the tracery drawn by the principle illustrated in Prob. 34. All curves that are contained by planes may be drawn in perspective in a similar manner, *e.g.* the patterns on floors, walls, and ceilings.

To make a Bird's-eye View.—Bird's-eye views are simply perspective views drawn with a very high horizon line.

Let Fig. 182 represent the plan of a district, drawn to a *scale of* 1 *inch* = 400 *feet*. We wish to make a bird's-eye view of this district, as it would appear from a point 1500 feet due south of the point A, and 500 feet above the ground plane [2].

Draw a horizontal line P.P. through A for the position of the picture plane. Draw a vertical line at A, and mark off along it, from A, equal distances of any convenient length (say 250 feet), and draw horizontal lines through them. Repeat the same distances on each side of A, and draw vertical lines through them to form squares, as shown.

We will draw the bird's-eye view to twice this scale, viz. 1 *inch* = 200 *feet*.

Draw a horizontal line G.L. (Fig. 183), and in any convenient position upon it mark the point A. Draw a vertical line at this point, $2\frac{1}{2}$ inches high (500 feet), to find the point C.V. [14]. Draw a horizontal line (H.L.) through C.V. for the level of the spectator's eye. As we have not room to find the point of distance (1500 feet), we will take one-third of this distance from C.V. along H.L. for $\frac{1}{3}$ D.P. (p. 59).

Set off on G.L., to the right of A, the points a and b, 250 feet apart, and join them to C.V.

We now have to find eight receding squares, which we will determine as follows :—

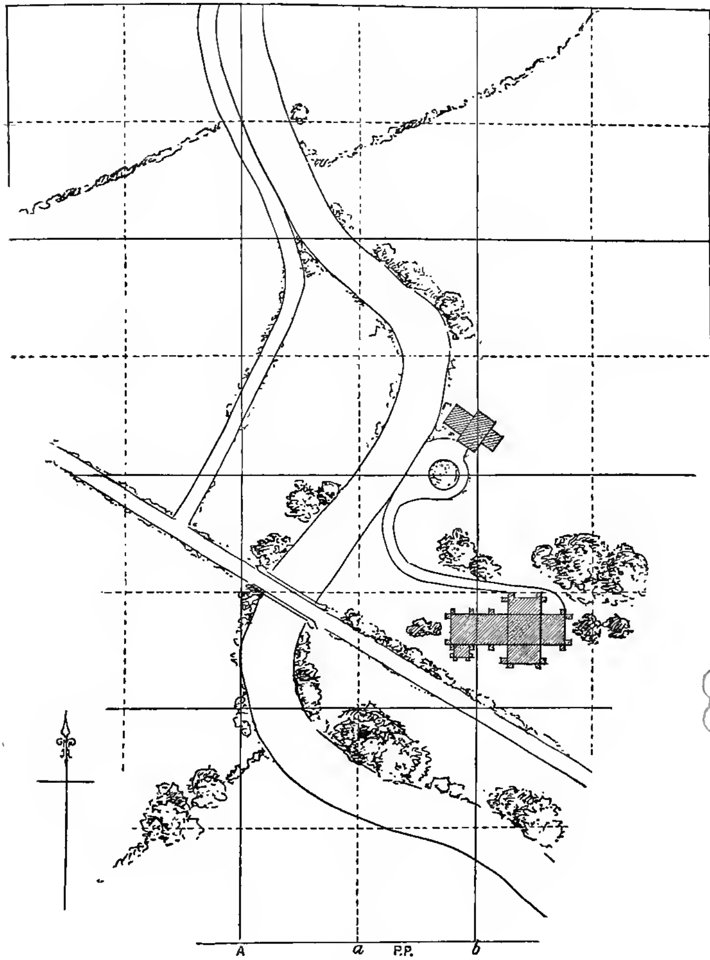


Fig. 182

Set off A*c*, equal to one-third of A*a*, and join *c* to C.V. Draw a line from *c* to $\frac{1}{3}$ D.P., to meet the receding line from A in *d*. Draw a horizontal line through *d* for the first row of

receding squares, and proceed in the same manner till the eight rows are determined.

The extra number of squares in the width of the back of the plan are determined by repeating the width of one of the back divisions on the last line, and drawing lines from C.V. through the points, as shown.

Having determined all these squares in perspective, we can

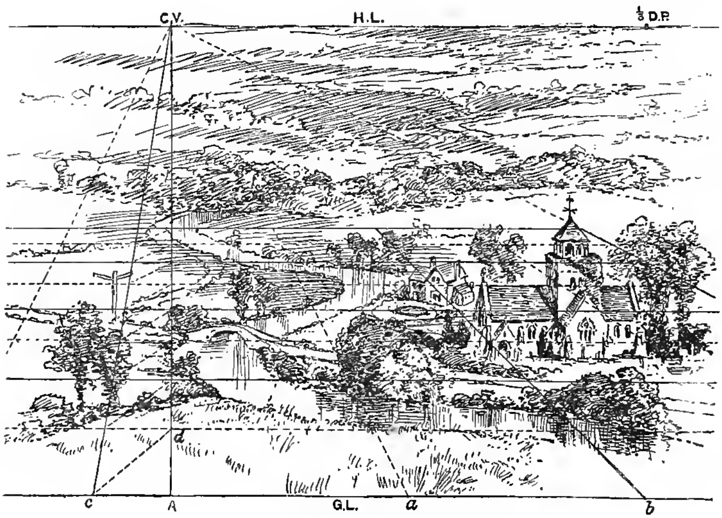


Fig. 183

draw the several lines and curves of the plan through the corresponding divisions of the perspective squares, and complete the bird's-eye view.

The vanishing points of the several horizontal lines of the buildings are determined by producing the receding lines in perspective to meet the H.L. [20].

The Centrolinead

The centrolinead is an instrument used for drawing lines converging towards inaccessible vanishing points, and is

indispensable when perspective drawings to a large scale are required to be made.

The ordinary form of the instrument is shown in Fig. 184. It consists of a ruling arm, D, attached to which are two movable arms, A and B, working on the same centre, and

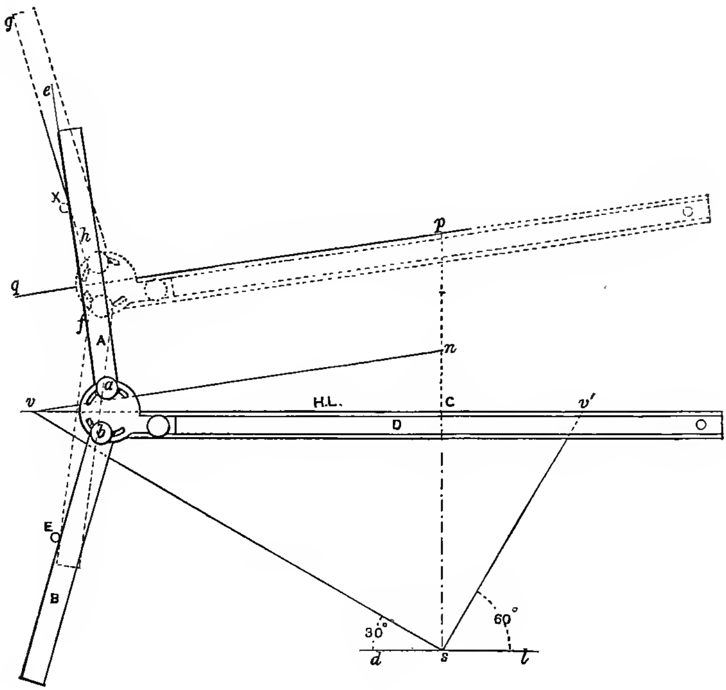


Fig. 184.

secured in the required position by the screws *a* and *b* working in slots. These movable arms work against studs fixed in the drawing board.

How to Set a Centrolinead.—Let H.L. (Fig. 184) represent the horizon line, and C the centre of vision. The station point to be $7\frac{1}{2}$ feet from C, and the *scale* $\frac{1}{2}$ *an inch* = 1 *foot.*

Take any aliquot part of $7\frac{1}{2}$ feet—say one-third ($2\frac{1}{2}$ feet)—

and set it off on a vertical line from C to find *s*. Draw a horizontal line *dl* through *s* [31].

Let the object we wish to draw form angles with the picture plane of 60° on the right, and 30° on the left. Set off these angles at *s* with *dl*, and draw lines through the points to meet H.L. in the points *v* and *v'*.

Draw a vertical line at C and upon it assume the position of any point *n*. Join *nv*. Repeat the distance C*n* three times from C to find *p*. Draw a line from *p*, parallel to *nv*, as *pq*. Then *pq* produced would meet H.L. in the true vanishing point for 30° on the left.

Open the arms A and B of the centrolinead to any convenient angle and secure them in position by the screws *a* and *b*. Place the upper edge of the ruling arm D against H.L. Fix the stud E in any convenient position against the left-hand side of the arm B, and rule a line *ef* against the left-hand side of the arm A. Now place the upper edge of the ruling arm A against the line *qp*, keeping the edge of the arm B against the stud E. The instrument will then be in the position shown by the dotted lines. Rule another line, *gh*, against the left-hand side of the arm A to intersect the line *ef* in X. Then X is the position of the second stud.

Note.—If the two lines drawn against the left-hand side of the arm A do not intersect, the angle between the arms A and B must be altered.

All lines ruled against the upper edge of the ruling arm D, when the arms A and B are in contact with the studs E and X, converge towards the same point.

CHAPTER XVI

PHOTOGRAPHIC PERSPECTIVE

Note.—Numerals in brackets [ ] refer to the numbered articles in Chapter II.

Photographs are now very much used by artists, especially "snap-shot" or instantaneous photographs, as by their assistance passing effects and movements can be recorded in a manner impossible by any other means. It is well known that the perspective in photographs of this kind is very much exaggerated on account of the use of lenses of short focus. The necessity for using such lenses for this class of work arises from the fact that they possess great depth of focus. This is very important for instantaneous work, as it is not possible with "snap-shot" photographs—generally taken with a hand camera —to focus the subject accurately before photographing it. *Depth of focus* or *depth of definition* are practical terms used by photographers; *e.g.* if one plane of the subject is sharply focused, all other objects, nearer or farther from that plane, will appear more or less wanting in sharpness, but the more depth of focus a lens has, the greater will be the distances focused on each side of the plane, in which objects will appear comparatively sharp in definition.

The depth of focus possessed by a lens varies inversely as the square of its focus; *e.g.* a lens of 6-inch focus has four times the depth of one of 12-inch focus.

The object of this chapter is to show how the exaggerated

perspective of a photograph may be remedied in making a drawing from it.

In addition to exaggerated perspective, distortion is also produced by the inherent property of some lenses, called *curvilinear distortion*; *i.e.* straight lines near the margin of the subject become curved in the photograph; such distortion is produced, more or less, by all single lenses, by portrait lenses, and the defect also occurs in some doublets.

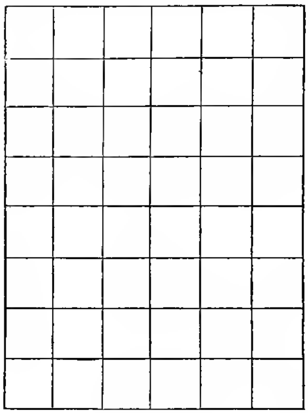


Fig. 185

The curvilinear distortion of a single lens varies with the position of the diaphragm.

If we were to draw a series of squares on a sheet of cardboard, as shown in Fig. 185, and photograph it by means of a single lens, with a diaphragm placed, in its usual position, in front of the lens—care being taken that both the cardboard and the sensitised plate should be in a vertical position—the photograph, if taken the full size of the plate, would appear similar to Fig. 186—"barrel-shaped"—in which all the lines, with the exception of those passing through the centre, are curved. But if the diaphragm were placed behind the lens, and then a photograph taken of Fig. 185, the full size of the plate, the result would be similar to Fig. 187—"pin-cushioned"—in which the lines passing through the centre would be straight, as before, but all the other lines

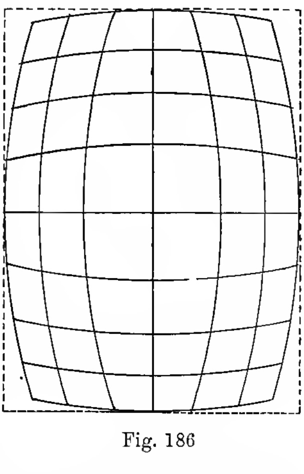


Fig. 186

would be curved in an opposite direction to those in Fig. 186.

In a proper constructed "doublet," the diaphragm being placed between the two single lenses, the back lens counteracts the curvature of the front lens, and the result should be as shown in Fig. 185.

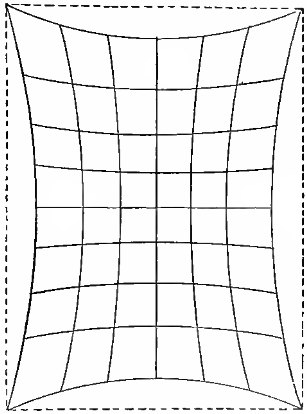


Fig. 187

The cause of exaggerated perspective, produced by a lens of short focus, or wide-angle lens, arises from the fact that the angle of view embraced by the lens is greater than the angle of distinct vision [44]. Some are made to include an angle of 90°, and some even a greater angle, and objects photographed by lenses of this character are affected in a similar manner to the cubes illustrated in Fig. 34, as fully explained in Article 45.

This exaggerated perspective could, to some extent, be avoided in many instances by using a lens made to cover a plate a size larger than the one required, but in doing so, we have to sacrifice a great deal of the subject taken from a given position, as the detail of the photograph would come out to a larger scale.

If an artist takes his own photographs, he can correct the curvilinear distortion of the photograph, when making a copy from it, whether produced by a single lens, portrait lens, or improperly constructed doublet, by the following simple contrivance.

Make a drawing of Fig. 185, to rather a large scale, with white lines on a dark ground, and photograph it to the full size of the plate. The negative, when developed and fixed, should show dark lines on a transparent surface.

On the drawing we wish to make from the photograph, draw the same number of equal squares in pencil, as shown in Fig. 185, to the scale required. Place the transparent negative, with the distorted lines, over a photograph taken with the same lens, and where the various parts of the photograph intersect the various lines of the transparent negative, draw corresponding intersections with the pencil lines on the drawing. The drawing will then be in true perspective, providing the angle of the lens does not exceed 60°.

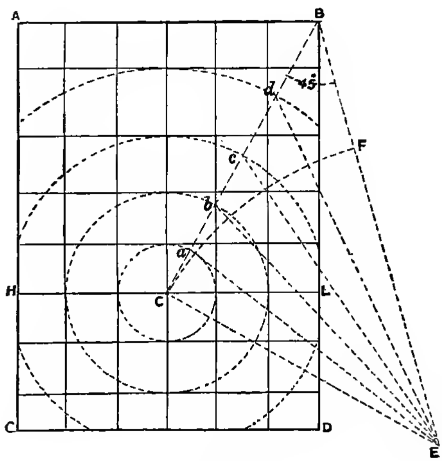


Fig. 188

The exaggerated perspective caused by a lens of short focus could be rectified in a drawing from it in the following manner :—

Let us assume the angle of the lens to be 90°. Draw on a sheet of paper a rectangle ABDG (Fig. 188), equal in size to the photograph, and mark the position of the horizon, as HL [18]. The point C is a point directly opposite the lens [14]. Join C to the corner of the rectangle farthest from it (B). Draw a line from C, at right angles to CB, as CE. Set off an angle of 45° at B, with BC, and produce the line to meet CE in E. Then BEC is half of the angle of the lens. With E as centre, and radius EC, draw an arc to meet EB in F.

Divide this arc into any number of equal parts (5), and draw lines from E through these divisions to meet the line CB in the points *a, b, c, d.* With C as centre, and the distances of these points from C as radii, draw circles. Draw lines, parallel to the sides of the rectangle, forming tangents to these circles.

These lines should be traced on a sheet of gelatine, or, what is better, on an unexposed plate, first rendered transparent by being immersed in a bath of hyposulphite of soda and then well washed and dried.

The transparent plate with these lines traced upon it should then be placed over a photograph taken with the lens. The lines could be drawn directly on the photograph, if desired, but by drawing them on a transparent plate they are available for any photograph taken with the same lens, and the photograph is not injured.

Let the rectangle ABDG (Fig. 189) represent our drawing.

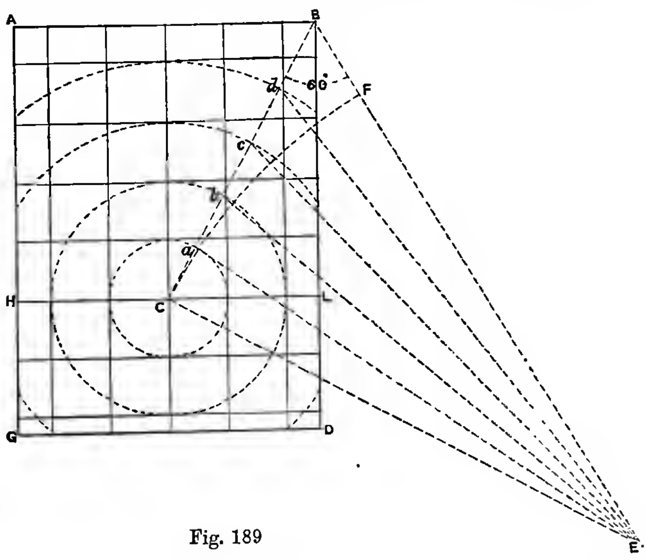


Fig. 189

This may be drawn to any scale to suit the size of the drawing, but the sides of the rectangle should have the same proportion

to each other as the corresponding sides of the rectangle (Fig. 188). The line HL, and the point C, should also be in corre-

Fig. 190

sponding positions to those shown in Fig. 188. Join CB, as before, and draw a line from C at right angles to CB, as CE. Set off an angle of 60° at B with BC and produce the line to meet CE in E.

Then BEC is equal to half of the angle of distinct vision [44].

Fig. 191

With E as centre, and EC as radius, draw the arc CF, and divide

it in five equal parts, as before, and find the points *a, b, c, d.* Complete the figure in a similar manner to that described for the preceding figure, which has corresponding letters. Make the different parts of the drawing intersect the various lines of Fig. 189, in accordance with the intersection of the photograph with the corresponding lines of Fig. 188.

If the artist does not take his own photographs, and wishes to make a drawing from one, he will first have to consider the angle subtended by the lens, judging from the amount of exaggeration in the perspective. This may be judged approximately, which is not difficult for a practised eye.

Fig. 190 is taken from a photograph, the angle subtended by the lens being 90°. The parallel lines forming the rectangles into which the view is divided are tangents drawn to circles, the radii of which are determined on the line CB, in a similar manner to those described in the construction of Fig. 188. In this view the perspective is very much exaggerated, and the receding distances appear much longer than they are in reality. The church appears considerably farther from the spectator than it really is.

Fig. 191 represents the same view with the perspective corrected. The parallel lines forming the rectangles into which the drawing is divided are tangents drawn to circles, the radii of which are determined on the line CB, in a similar manner to those described in the construction of Fig. 189.

THE END

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